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Equation of state of asymmetric nuclear matter: a VMC study

Research Article

Kaan Manisa[1]*, Ülfet Atav[2], Sibel Sarıaydın[1]

1 *Dumlupinar University, Physics Department, Kütahya, Turkey*

2 *Selçuk University, Department of Physics, 42079, Konya, Turkey*

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Abstract: A Variational Monte Carlo (VMC) method is employed to investigate the properties of symmetric and asymmetric nuclear matter. The realistic Urbana V_{14} twonucleon interaction potential of Lagaris and Pandharipande was used to describe the microscopic interactions. Also, many body interactions are included as a density dependent term in the potential. Total kinetic and potential energies per particle are calculated for asymmetric nuclear matter by VMC method at various densities and isospin asymmetry parameters. The results are compared with data found in literature, and it was observed that the results obtained in this study reasonably agree with the results found in the literature. Also, the symmetry energy and incompressibility factor of the nuclear matter were obtained. The results obtained are in good agreement with those obtained by various authors with different methods and techniques.

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1. Introduction

Nuclear matter is a uniform hypothetical system with translational invariance and has a fixed ratio of neutrons and protons (ignoring the Coulomb forces). When the number of protons and neutrons are the same, the system is called symmetric nuclear matter (SNM). Many calculations have been performed using different methods to solve the many-body problem of infinite symmetric nuclear matter and to describe the saturation and bulk properties of SNM [1–8].

*E-mail: kmanisa@dumlupinar.edu.tr

The properties of asymmetric nuclear matter (ASNM) have also been studied using both the non-relativistic models [9–12] and relativistic models [13–15]. The ASNM calculations, found in the literature, have employed various approaches such as (i) the non relativistic and self-consistent Dirac-Brueckner-Hartree-Fock schemes [2, 8] (ii) the variational methods [12, 13, 16], (iii) the relativistic mean field and chiral sigma models [17, 18], (iv) the phenomenological Skyrme interaction [7, 19] and (v) the Green function formalism [20].

These calculations are useful because they can provide an equation of state (EOS) for the nuclear matter which is needed for studying problems such as relativistic heavy-ion collisions or neutron star structures [21]. The investigation of ASNM is also of importance for astrophysical

studies such as the physics of supernova explosions [22]. The theories of neutron-rich nuclei [23–25], and their collisions [26] also require a knowledge of the asymmetric nuclear EOS. Very sophisticated variational calculations on the properties of the pure neutron matter (PNM) and symmetric nuclear matter (SNM) were performed by Akmal *et al.* using a variational hypernetted chain approach for various realistic nucleon-nucleon (NN) interactions. Akmal *et al.* have not calculated the properties of the ASNM directly, instead they have used a quadratic interpolation to determine the properties of ASNM for the proton fractions between the PNM and SNM limits.

Recently, we have obtained ground state properties of SNM by using a Variational Monte Carlo method (VMC) [1]. Here we extended that study to obtain the EOS and some other thermodynamic properties of ASNM. The NN interaction potential we used in this study is an earlier version of one of the potentials used by Akmal in their Variational Hypernetted Chain Study.

The first application of the Variational Monte Carlo methods (VMC) in quantum mechanical systems was looked at by McMillan [27] investigating bosonic systems. VMC methods were extended to study simple Fermi systems [28, 29] and were developed for finite nucleon systems, and these methods have successfully been applied to few nucleon systems [30, 31]. Saturation of ASNM is a fundamental nuclear property which dominates the properties of unstable nuclei under astrophysical conditions. While the saturation density and energy of SNM are determined empirically from masses and radii of stable nuclei [32], the properties of ASNM are still uncertain. In this paper, we obtain the saturation energies of asymmetric nuclear matter by the VMC method for various isospin asymmetry parameters, β, and at various densities. We calculate saturation density and energy of ASNM as functions of isospin asymmetry parameter, β. We use the realistic Urbana V14 potential for the nucleon-nucleon interaction and we include the three-body interactions through a density dependent term.

2. VMC calculations of asymmetric nuclear matter

2.1. Interaction potential

The Hamiltonian operator of a free system of N particles interacting through a two body interaction potential V_{ij} can be written as

$$\mathbf{H} = -\frac{\hbar^2}{2m}\sum \nabla_i^2 + \sum_{i<j} V_{ij}. \quad (1)$$

In principle, one can determine all the ground state properties of such a system if the two body interaction potential is known. However, the exact nature of the nucleon-nucleon interaction is still unknown. There are many realistic model potentials proposed to describe the nucleon-nucleon interaction. Parameters of these realistic potentials are determined so as to obtain correct binding energy for deuteron and by fitting to phase shift data obtained in nucleon scattering experiments. The phase-shift data varies greatly from channel to channel and it is necessary to have operator components. In this paper we use Urbana V_{14} potential, which was proposed by Lagaris and Pandharipande, and contains 14 operator components. The parameters of the potential were obtained by fitting the phase-shift data from low energy nucleon-nucleon scattering experiments and the properties of the deuteron [10, 11]:

$$V_{ij} = V^c + V^\sigma(\sigma_i\cdot\sigma_j) + V^\tau(\tau_i\cdot\tau_j) + V^{\sigma\tau}(\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j) + V^t S_{ij} + V^{t\tau} S_{ij}(\tau_i\cdot\tau_j) + V^b(\mathbf{L}\cdot\mathbf{S})_{ij} + V^{b\tau}(\mathbf{L}\cdot\mathbf{S})_{ij}(\tau_i\cdot\tau_j)$$
$$+ V^q L^2 + V^{q\sigma} L^2(\sigma_i\cdot\sigma_j) + V^{q\tau} L^2(\tau_i\cdot\tau_j) + V^{q\sigma\tau} L^2(\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j) + V^{bb}(\mathbf{L}\cdot\mathbf{S})^2 + V^{bb\tau}(\mathbf{L}\cdot\mathbf{S})^2(\tau_i\cdot\tau_j). \quad (2)$$

Due to the translational invariance of the infinite nuclear matter, the terms depending on the relative angular momentum operator $\vec{\mathbf{L}}$, do not considerably effect the binding energy. Furthermore, the contributions of latter terms are much smaller than those of the first four, while their evaluation requires much more numerical effort thus rendering the problem intractable. Therefore only the first four terms of the Urbana potential were retained in the calculations of expectation value. Thus, we have

$$V_{ij} = V^c + V^\sigma(\sigma_i\cdot\sigma_j) + V^\tau(\tau_i\cdot\tau_j) + V^{\sigma\tau}(\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j), \quad (3)$$

for the two-body interaction. Where V^c, V^σ, V^τ, and $V^{\sigma\tau}$ depend only on the distance between the nucleons i and

Table 1. Parameters of the Urbana V_{14} nucleon-nucleon potential.

I	I^i	S^i
C	-5.7030	2575.3
σ	0.7628	-366.56
τ	0.8892	-466.56
$\sigma\tau$	-0.2790	402.81

c=0.2 fm^{-2}, R=0.5 fm, a=0.2 fm

j. In the Urbana potential each term in Eq. (2) has three parts

$$V^i = V^i_\pi + V^i_I + V^i_S, \tag{4}$$

representing long-range (V^i_π), intermediate-range (V^i_I), and short-range (V^i_S) interactions. The long range part of the interaction (V^i_π) is nonzero only for $i = \sigma\tau$ and is given by

$$V^{\sigma\tau}_\pi = 3.488\frac{e^{-\mu r}}{\mu r}(1 - e^{-cr^2}), \tag{5}$$

where $\mu = 0.7$ fm^{-1} is the inverse compton wavelength for pions. The intermediate and short range parts are

$$V^i_I(r) = I^i\left[\left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right)\frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right)\right]^2, \tag{6}$$

and

$$V^i_S(r) = \frac{S^i}{1 + e^{(r-R)/a}} \tag{7}$$

respectively. Values of the potential strengths I^i and S^i and the parameters c, R, a were given by Lagaris and Pandharipande [10, 11] and are shown in Tab. 1.

The three and more body interactions should be incorporated into any consistent nuclear matter calculation. Because, none of the nuclear matter studies by various two-body interactions [3, 6, 7, 10, 11] was solely able to explain all the properties of nuclear matter. Also, all two-nucleon interaction models estimate too large an equilibrium density for nuclear matter. The observations of overbinding at high densities suggest that the short ranged repulsive part of two-body interactions should be more effective at high densities. Keeping these considerations in mind, the three (and more) body interactions are represented by density dependent terms in the potential found in many other works of literature [13, 33]. The density dependent term contains a few free parameters, which are adjusted to obtain the correct binding energy and saturation density of the SNM. This approach is solely phenomenological and is not satisfactory on a rigorous basis. A better description of the three body interactions might be obtained by fitting parameters of some potential model (for example two pion exchange, meson exchange) to the data from the binding energies and form factors of light nuclei and to the saturation point of symmetric nuclear matter. Even in that case a phenomenological potential term might be necessary to produce correct saturation behaviour of the symmetric nuclear matter as in Urbana IX three nucleon interaction [34]. However, using such a potential is practically forbidding for the calculations reported in this study. Therefore, we use the phenomenological approach assuming the density dependent term to be proportional to short ranged part of the Urbana potential and we assume that the total interaction, including the many body effects, is of the form

$$v_{14} + TNI = v_\pi + v_I + v_s + v_s(\alpha\rho)^\gamma, \tag{8}$$

where ρ is the number density of nucleons. α and γ in the above equation are free parameters and adjusted so as to obtain the correct binding energy and saturation density of SNM.

2.2. The variational Monte Carlo method

In the variational Monte Carlo calculations to obtain the properties of bulk nuclear matter we consider a cubic box of side L containing N nucleons with periodic boundary conditions. The trial wave function used in the present study is a Jastrow type wave function in the form

$$\Psi_j\left(\vec{R}\right) = \prod_{i<j} f_j(r_{ij})\Phi, \tag{9}$$

where Φ is the many particle wave function for the system of non-interacting particles and $\vec{R}$ is a 3N dimensional vector representing the coordinates of particles, while f_j is the two particle correlation function. Jastrow suggests that this correlation function in general will be an operator function [35]. However in most applications f_j is assumed to depend only on the interparticle distance, $r_{ij} = |r_i - r_j|$. In fact for the nuclear matter the interaction potential is operator dependent and so are the correlation functions. Employment of such an operator dependent correlation function would have required an overload of numerical effort making the problem unfeasible within our computational facilities. Therefore we have assumed the correleation function to depend only on the interparticle distance.

One can use plane waves $\phi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$ for the single particle wave functions of the nucleons in bulk matter. Because we consider nucleons restricted to a cubic box of side L,

$\vec{k} = 2\pi\vec{n}/L$ and $\vec{n}$ is an integer vector. In order to conserve the rotational invariance of bulk nuclear matter we restrict the number of neutrons and protons in the box to completely filled shells only. We assume that the space and spin parts of the wavefunction is separable. Under these conditions choosing a many particle trial wave function with

$$\Phi(R) = D^{P\uparrow} D^{P\downarrow} D^{N\uparrow} D^{N\downarrow} \tag{10}$$

is quite reasonable because the spin-isospin dependent parts of the interaction potential is relatively weak. The spin-isospin exchange term might be expected to have a small but significant effect on the form of the wave function, but we prefer the form in Eq. (10), due to some technical problems arising from the method we have used in this study. Also we have performed calculations for only fully occupied closed shells to minimize these affects. Also it is well known that the expectation value of the total energy is not very sensitive to small changes in the trial wave function.

The determinants $D^{P\uparrow}$, $D^{P\downarrow}$, $D^{N\uparrow}$ and $D^{N\downarrow}$ in Eq. (10) are the slater determinants of single particle wave functions for corresponding spin, isospin state then

$$D^s = \det(d^s_{ij}), \tag{11}$$

where

$$d^s_{ij} = \phi_j((\vec{r}, s)_i). \tag{12}$$

The nuclear forces are short ranged and saturates very quickly, thus the radial distribution function is not expected to have very long range correlations therefore for the two particle correlation function f_j in Eq. (9) we use a function in the form

$$f_j(r) = \left[\frac{1}{1 + e^{(r_0 - r)/a}}\right]^t, \tag{13}$$

where t, r_0 and a are variational parameters. We define a pseudo potential $u(r)$ for practical reasons such that $f_j(r_{ij}) = \exp(-u(r_{ij}))$ then our variational wave function becomes

$$\Psi_j = \exp\left(-\sum_{i<j} u(r_{ij})\right) D^{P\uparrow} D^{P\downarrow} D^{N\uparrow} D^{N\downarrow}. \tag{14}$$

We sample the 3N dimensional space with the probability distribution

$$\frac{|\Psi(R)|^2}{\int dR\,|\Psi(R)|^2} \tag{15}$$

using a random walk created by the usual Metropolis method. The method given above is a slightly modified version of the VMC method for fermions defined by Ceperley *et al.* [28]. They have also discussed in detail the use of a trial wave function of this form.

The expectation value of any operator F is then simply the average value of the operator evaluated for the coordinates of the random walk with M moves

$$\left\langle \hat{F} \right\rangle = \frac{\int d\vec{r}\Psi^*(\vec{r})F(\vec{r})\Psi(\vec{r})}{\int d\vec{r}\,|\Psi(\vec{r})|^2} \simeq \frac{1}{M}\sum_{i=1}^{M} F(\vec{r}_i). \tag{16}$$

Total energy of the system is calculated using this approach. The contribution of the nucleon-nucleon interactions to total energy are calculated for interparticle separations up to a cut off distance of $L/2$. A reasonable approximation to include the contributions of the pairs farther apart is to assume that the density of particles is constant outside this interaction sphere.

For each density and asymmetry parameter the total energy corresponding to the Hamiltonian of the system is calculated for various values of the parameters in the trial wave function. Then the variational parameters r_0, a, and t are determined from these calculations so that the total energy is a minimum. Then a final Monte Carlo calculation of the system with the optimized parameter set is performed.

As we have mentioned before, one must use fully occupied closed shells of plane waves for both neutrons and protons in order to preserve the isotropy of the system. Thus the number of neutrons or protons must be chosen from the set (2, 14, 38, 54, 66, 114, ...). The isospin asymmetry parameter, β, is defined as $\beta = \frac{N_n - N_p}{N_n + N_p}$ where N_n and N_p are the numbers of neutrons and protons in the cubic box under consideration.

3. Results

In this section, we present the results obtained from Monte Carlo simulations for ASNM with various isospin asymmetry parameters. The parameters for the phenomenological three nucleon interaction were determined as $\alpha = 1.99$ and $\gamma = 2.07$ to obtain the correct saturation point of SNM. We have obtained the expectation values of the kinetic, potential and total energies per nucleon at densities between $\rho = 0.02$ fm^{-3} and $\rho = 0.20$ fm^{-3} in 0.02 steps for each isospin asymmetry parameter. From these data, we have obtained the saturation density, saturation energy, incompressibility factor and symmetry energy of ASNM.

3.1. The nuclear EOS

The dependence of the total energy per nucleon on the density of nuclear matter is called as the nuclear EOS.

Table 2. Expectation values of total energies per nucleon for ASNM obtained from VMC. The densities are in units of fm^{-3} and the energies are in MeV.

ρ \ β	0.0	0.1	0.17392	0.26924	0.35714	0.5	0.58824	0.75	0.78124	0.9	1.0
0.02	-0.53738	-0.46551	-0.49057	-0.17075	0.34967	0.99206	1.11562	2.40431	2.96696	3.46915	4.85248
0.04	-3.27244	-3.27045	-3.32332	-2.76952	-2.02853	-0.54635	-0.42283	1.66964	2.66473	3.48098	6.30358
0.06	-6.16522	-6.25625	-6.43074	-5.58774	-4.58167	-3.01000	-2.47180	0.43916	1.84853	3.17467	7.37197
0.08	-9.31009	-9.39223	-9.13495	-8.33957	-7.21064	-5.16595	-4.08827	-0.71879	0.85878	2.75656	9.16279
0.10	-12.08352	-11.95682	-11.97683	-10.74616	-9.37700	-6.99498	-5.91384	-1.75686	0.13380	2.30194	9.67951
0.12	-14.15952	-13.73828	-14.18132	-12.74092	-11.17105	-8.31464	-7.19832	-2.92666	-0.10244	2.74579	11.3466
0.14	-15.70309	-15.52161	-15.65634	-11.55512	-11.56693	-9.00946	-7.73176	-1.71456	0.49364	3.29812	12.84952
0.16	-16.06530	-16.01391	-16.31325	-13.76260	-12.17436	-8.73832	-7.42302	-1.43002	1.73442	4.74846	15.2078
0.18	-15.55622	-15.52853	-15.61907	-13.35359	-11.40420	-7.52419	-6.10432	1.11956	7.94602	7.13643	17.82158
0.20	-13.79394	-13.97431	-13.84490	-11.91865	-9.12039	-5.13147	-3.59740	4.06757	7.09619	10.80490	21.18701

Table 3. Expectation values of potential energies per nucleon for ASNM obtained from VMC. The densities are in units of fm^{-3} and the energies are in MeV.

ρ \ β	0.0	0.1	0.17392	0.26924	0.35714	0.5	0.58824	0.75	0.78124	0.9	1.0
0.02	-7.01241	-7.04839	-6.94595	-6.90959	-6.72042	-6.39650	-6.17204	-5.71213	-5.44570	-5.04730	-4.67126
0.04	-13.87671	-13.75751	-13.81512	-13.58107	-13.24744	-12.73003	-12.21302	-10.85585	-11.34001	-10.34640	-9.33267
0.06	-21.05492	-20.10072	-20.31318	-19.45508	-19.76251	-18.99661	-18.08302	-15.42282	-16.44054	-14.45697	-13.40202
0.08	-27.12615	-26.67887	-24.60298	-25.10494	-25.98200	-24.62568	-23.34772	-19.65058	-21.48076	-18.74775	-14.82448
0.10	-32.76589	-31.04087	-32.16786	-30.25635	-30.90420	-29.88638	-27.13183	-23.08741	-25.45376	-23.34240	-19.95867
0.12	-38.13538	-36.25653	-35.22762	-34.85368	-35.65934	-34.86439	-31.74140	-24.48167	-29.52520	-24.84279	-23.50307
0.14	-41.71833	-42.06813	-40.03235	-43.80768	-39.17222	-38.43529	-36.18789	-28.49691	-32.66903	-28.90254	-26.49182
0.16	-46.67880	-43.97848	-44.42552	-40.76109	-43.23779	-41.38881	-37.53172	-29.89903	-36.60538	-29.50655	-27.69502
0.18	-47.95201	-48.34382	-44.66007	-42.67222	-45.24184	-43.66842	-39.35836	-29.26248	-42.50700	-31.84768	-30.42782
0.20	-48.27466	-48.61877	-48.49524	-44.96696	-46.43844	-45.53113	-41.45469	-27.30471	-37.13382	-26.72804	-32.79321

Table 4. Expectation values of kinetic energies per nucleon for ASNM obtained from VMC. The densities are in units of fm^{-3} and the energies are in MeV.

ρ \ β	0.0	0.1	0.17392	0.26924	0.35714	0.5	0.58824	0.75	0.78124	0.9	1.0
0.02	6.50227	6.61525	6.50631	6.77932	7.08588	7.40666	7.35630	8.42574	8.43559	8.61692	9.52374
0.04	10.77769	10.68487	10.77117	11.04463	11.33120	12.30649	12.12752	13.58702	14.13079	14.24759	15.63625
0.06	15.33969	14.34772	14.55770	14.44383	15.48812	16.31519	16.38587	17.96484	18.61420	18.53094	20.77399
0.08	18.65916	18.21813	16.67913	17.81404	19.36645	20.08724	20.60687	22.32074	22.94668	23.00924	23.98727
0.10	22.02114	20.55068	22.05689	21.14166	22.49431	23.90131	23.25187	26.22959	26.54976	27.86436	29.63818
0.12	25.90128	24.61431	23.67047	24.42387	25.90358	28.01698	27.36249	28.17363	30.80506	30.62329	34.84966
0.14	28.60903	29.35637	27.85061	35.32968	29.53803	31.41821	32.14993	35.31694	35.02372	36.14349	39.34134
0.16	33.94989	31.56473	32.52080	30.91991	33.57705	35.22998	34.75838	39.10248	40.73322	39.19510	42.90282
0.18	36.54295	37.27600	34.46020	34.15624	36.99071	39.36798	38.93555	43.28269	53.42842	45.00734	48.24940
0.20	39.50185	40.03084	41.15163	38.86895	41.16503	44.32078	44.64228	46.69220	47.83372	44.72234	52.06010

Table 5. Values of saturation densities and energies of asymmetric nuclear matter for various isospin asymmetry parameters.

β	Saturation Density ρ_0 (fm^{-3})	Saturation Energy E_0 (MeV)
0	0.1585	-15.83
0.1	0.1610	-15.69
0.17392	0.1593	-15.88
0.26924	0.1558	-13.80
0.35714	0.1493	-11.91
0.5	0.1397	-8.86
0.58824	0.1381	-7.56
0.75	0.1204	-2.65
0.78124	0.1114	-0.164
0.9	0.1004	2.20

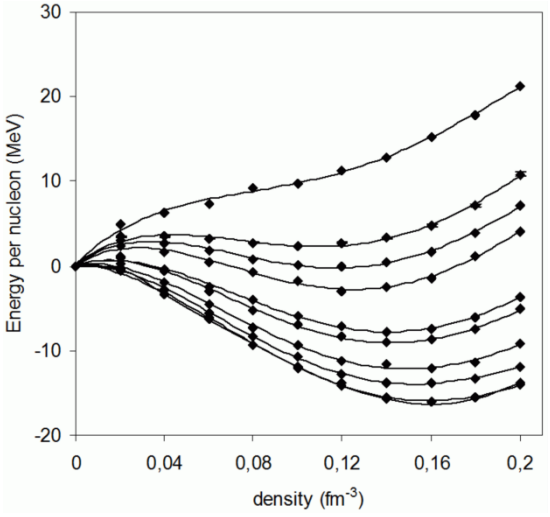

Figure 1. The results for the EOS of asymmetric nuclear matter for isospin asymmetry parameters β=0.0, 0.1, 0.17392, 0.26924, 0.35714, 0.5, 0.58824, 0.75, 0.78124, 0.9, 1.0. The β values increase from bottom to top with the lowermost curve corresponding to SNM (β=0.0) and the uppermost one corresponding to PNM (β=1.0).

The kinetic, potential and total energies per nucleon obtained from the Monte Carlo simulations are presented in Tabs. 2, 3 and 4. Fig. 1 shows the total energies per nucleon as a function of density for various values of the isospin asymmetry parameter. The lowermost curve corresponds to the SNM (β=0.0) and the uppermost one correspond to pure neutron matter (PNM, β=1.0). It can be clearly seen from the figure that the saturation energy increases with increasing isospin asymmetry parameters while the saturation densities of ASNM decrease with isospin asymmetry. We have obtained the saturation energies and densities of ASNM for each isospin asymmetry parameter and the results are presented in Tab. 5. Saturation energy almost vanishes for isospin asymmetry parameters around β=0.8. For β values greater than 0.8 the saturation energy becomes positive indicating that the ground state of ASNM is not a bound state and therefore it tends to expand in the absence of an external confining potential. For example, when β=0.9, the saturation energy of ASNM is obtained to be E_0=2.2 MeV. Also it can be seen from Fig. 1 that there is no saturation point for PNM (β=1.0).

The behaviour of the EOS of nuclear matter is consistent with the previous studies based on phenomenological nucleonnucleon interaction models. For example Fig. 2, shows our results for saturation properties of ASNM along with results from a recent study of EOS of the ASNM by Oyamatsu *et al.* [36] where both Skyrme HartreeFock calculations with SIII interaction [37] and the relativistic mean field calculations with TM1 parameter set [38] are performed to obtain the EOS of the ASNM.

The results obtained in this study are very similar to those obtained from TM1 parameter set for the saturation energy however there is a small discrepancy between the saturation densities. On the other hand, the saturation densities found in this study and the ones obtained from TM1 parameter set show almost the same behaviour. The small discrepancy can be attributed to the different reference values assumed for the saturation density of the SNM.

The method proposed in this study simplifies the simulation of nuclear matter problems to be solvable with ordinarily accessible computational capabilities, such as a personal computer. On the other hand our results are in reasonable agreement with the results of recent complicated supercomputer applications. For example, in a recent study, Carlson *et al.* [39] have investigated the properties of pure neutron matter with Variational Chain Summation, Variational Monte Carlo and Green Function Monte Carlo Methods using Argonne v^8 nucleon-nucleon and Urbana IX three nucleon interactions. Our results and the ones obtained by Carlson *et al.* for total energies per nucleon are given in Tab. 6. Our results for the total energy is somewhere between the GFMC and VMC calculations of Carlson *et al.*, indicating that the differences are reasonably small and can be mainly attributed to the use of different interaction potentials.

The Skyrme potential and other phenomenological potential models are, in fact, very convenient and useful in the calculations of the bulk properties of nuclear matter, however before using such potentials the reliability of the potential model should be established. Monte Carlo simulation results obtained in this study from the first principles

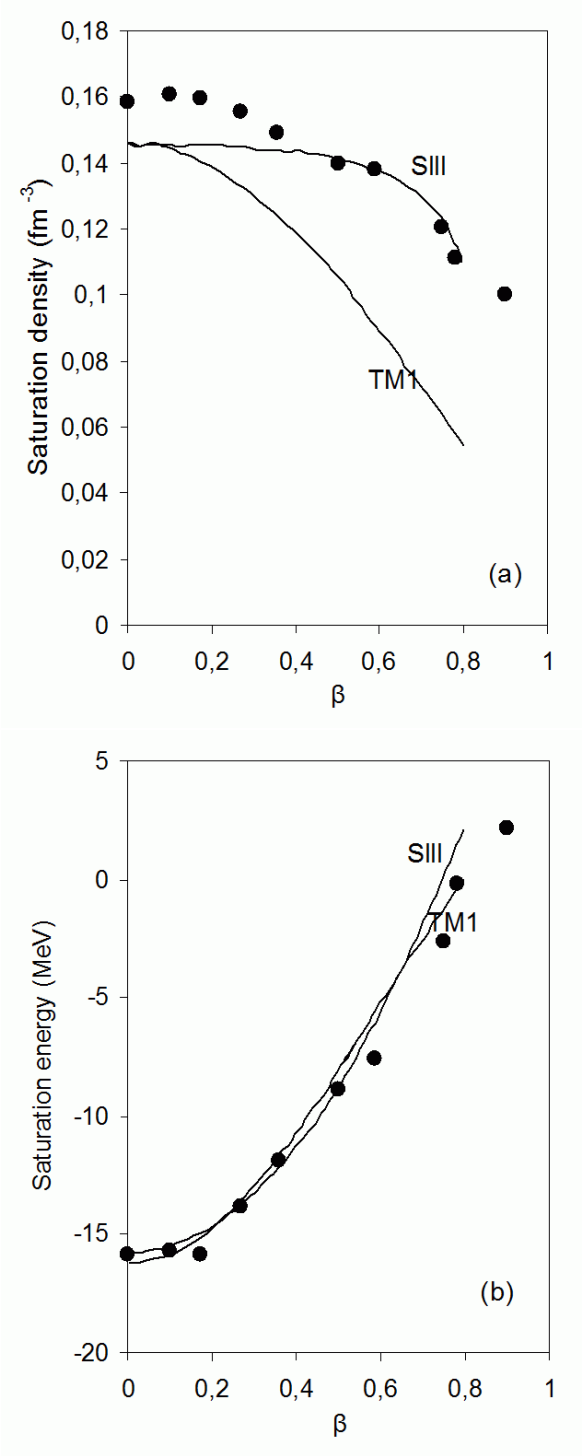

Figure 2. Saturation density (a) and energy (b) of asymmetric nuclear matter as functions isospin asymmetry parameter β obtained by Oyamatsu *et al*. along with the results of this study (solid disks). The label SIII indicates the Skyrme HartreeFock calculations with SIII parameter set and the label TM1 indicates the relativistic mean field calculations with TM1 parameter set.

Table 6. Total energies per nucleon of pure neutron matter. Last column shows the results of this study and E_{VCS}, E_{VMC} and E_{GFMC} are from Ref. [39].

ρ (fm^{-3})	E_{VCS} (MeV)	E_{VMC} (MeV)	E_{GFMC} (MeV)	E (MeV)
0.4	6.6	7.0	6.0	6.30
0.8	9.2	10.3	8.4	9.16
0.16	12.1	17.4	12.1	15.20

and a nucleonnucleon interaction potential may serve as a means to evaluate relative merits of various phenomenological models. Also the same data may be used to optimize the parameter sets of such potentials.

3.2. Symmetry energy and the incompressibility factor of ASNM

The energy per particle of asymmetric nuclear matter can be expanded about symmetric nuclear matter in a Taylor series in terms of the isospin asymmetry parameter β

$$E(\rho,\beta) = E(\rho) + \beta^2 S_1(\rho) + \beta^4 S_2(\rho) + \ldots, \qquad (17)$$

If we ignore the terms of order β^4 and higher then we can write the total energy per particle of ASNM as

$$E(\rho,\beta) = E_s(\rho) + \beta^2 S(\rho), \qquad (18)$$

where $E_s(\rho)$ is the energy per nucleon of SNM and $S(\rho)$ is called as the symmetry energy. When the density, ρ, is close to the saturation density, ρ_0, of SNM we can also expand $E_s(\rho)$ and $S(\rho)$ in Taylor series around, ρ_0, again retaining only the terms up to the order of $(\rho-\rho_0)^2$ we obtain

$$E_s(\rho) = E_0 + \frac{K_s}{18\rho_0^2}(\rho-\rho_0)^2, \qquad (19)$$

$$S(\rho) = S_0 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K}{18\rho_0^2}(\rho-\rho_0)^2. \qquad (20)$$

Here E_0 is the saturation energy of SNM and $S_0 = S(\rho_0)$ is the symmetry energy at the saturation density of SNM. K_s is the usual incompressibility of symmetric matter

$$K_s = 9\rho_0^2 \frac{d^2E_s}{d\rho^2}\bigg|_{\rho=\rho_0}, \qquad (21)$$

If the approximation in Eq. (18) is valid up to $\beta = 1$ then p_0 and $K_s + \Delta K$ correspond to the pressure and the incompressibility of pure neutron matter. However, strictly speaking p_0 and ΔK are just derivatives of the symmetry energy at the nuclear-matter density

Table 7. Symmetry energies and Saturation Energies of SNM at various densities.

Density ρ (fm^{-3})	Saturation Energy of SNM $E_S(\rho)$ (MeV)	Symmetry Energy $S(\rho)$ (MeV)
0.02	-0.51	5.26
0.04	-3.35	9.27
0.06	-6.51	13.06
0.08	-9.65	17.16
0.1	-12.36	20.23
0.12	-14.58	23.56
0.14	-15.59	26.09
0.16	-16.52	29.20
0.18	-15.92	32.50
0.2	-14.27	33.70

$$p_0 = \rho_0^2 \frac{dS}{d\rho}\bigg|_{\rho=\rho_0}, \tag{22}$$

$$\Delta K = 9\frac{d}{d\rho}\left[\rho^2 \frac{dS}{d\rho}\right]_{\rho=\rho_0}. \tag{23}$$

We evaluate symmetry energies and the incompressibility by fitting the expressions given above to the total energy data of ASNM presented in Tab. 2. Obtained symmetry energies are presented in Tab. 7. The values of the symmetry energy are consistent with those reported in the literature. The value obtained for the symmetry energy at the saturation density is 29.2 MeV and this coincides with the experimentally quoted value of 30 ± 4 MeV [40].

Fitting the obtained symmetry energy values to the relation

$$S(\rho) = C_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{24}$$

we find the parameters C_0 and γ as 29.27 MeV and 0.82 respectively which are close to the experimental values of 31-33 MeV and 0.55-0.69 quoted by Shetty *et al.* [41]. Also using the results of this fit and Eq. (23) one can obtain the ΔK parameter as -393.15 MeV which can be compared with the recent experimental value of -550 ± 100 MeV given by Li *et al.* [42]

Another property of the ASNM, the incompressibility factor, appears in some sophisticated mass formulas, however it cannot be precisely determined from these formulas and the quoted values in the literature have a wide range from 240 to 300 MeV with error estimates of ± 50 MeV. We have obtained the incompressibilitiy factor of SNM as 383.9 MeV, this value is somewhat different then the experimental value but considering the large error bars in the quoted experimental values it might be acceptable.

In general, the results obtained in this study are very close or overlap with both the experimental and the theoretical studies found in the literature. The enhancement of the short ranged repulsive part of the potential results in a larger incompressibility.

4. Conclusion

The equation of state of nuclear matter is of great interest in nuclear physics and astrophysics. EOS can be used to study nuclear fission, heavy ion reactions and hot neutron stars in nuclear physics and astrophysics. The EOS is also of fundamental importance in the theories of nucleus-nucleus collisions at energies where the nuclear incompressibility K comes into play as well as in the theories of supernova explosions. Therefore, the equation of state of nuclear matter has been studied extensively in non-relativistic approach and the relativistic mean field theory [43] by different many body methods. The different realistic models of nucleon-nucleon interaction obtained by fitting the nucleon-nucleon scattering data and deuteron properties are used in these methods [44].

Here we have presented the results of a VMC simulation of the ASNM using Urbana potential. The results are compared with the data found in the literature, and it was observed that the results obtained in this study reasonably agree with the results found in the literature. The small discrepancies between the results of this study and the experimental values can be eliminated through a refinement of the three nucleon interactions. Another important point is that the results presented in this study may serve as the results of a computer experiment.

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