T.C.
MANİSA CELAL BAYAR ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

YÜKSEK LİSANS TEZİ
MATEMATİK ANABİLİM DALI
TOPOLOJİ BİLİM DALI

ESNEK METRİK UZAYLAR ÜZERİNDE ESNEK KOMPAKT KÜMELERİ İÇİN EN YAKIN NOKTASI

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MANİSA–2023

TEZ ONAYI

Watson LUNG tarafından hazırlanan **"Esnek Metrik Uzaylar Üzerinde Esnek Kompakt Kümeleri İçin En Yakın Noktası"** adlı tez çalışması 05 /01/2023 tarihinde aşağıdaki jüri üyeleri önünde Manisa Celal Bayar Üniversitesi Fen Bilimleri Enstitüsü **Matematik Anabilim Dalı**'nda **YÜKSEK LİSANS TEZİ** olarak başarı ile savunulmuştur.

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Bu tezin Manisa Celal Bayar Üniversitesi Fen Bilimleri Enstitüsü Matematik Anabilim Dalı'nda, akademik ve etik kurallara uygun olarak yazıldığını ve kullanılan tüm literatür bilgilerinin referans gösterilerek tezde yer aldığını beyan ederim.

Watson LUNGU

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SİMGELER VE KISALTMALAR DİZİNİ

Φ	Boş Esnek Cümle
$\mathbb{N}$	Doğal Sayılar Cümlesi
$\prec$	Esnek Anlamda Önce Gelme
$\tilde{\mathfrak{G}}$	Esnek Metrik
$((\mathfrak{F},\mathfrak{P}),\tilde{\mathfrak{G}})$	Esnek Metrik Uzay
$\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$	Esnek Dizi
$(\mathfrak{F},\mathfrak{P})$	Esnek Cümle
$\tilde{\subseteq}$	Esnek Altcümle
$\tilde{\cup}$	Esnek Cümlelerde Birleşim
$\tilde{\cap}$	Esnek Cümlelerde Kesişim
$(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))$	Esnek Nokta
$(\mathfrak{b},\mathfrak{r})$	Esnek Parametrik Skaler
$(\mathfrak{K},\mathfrak{T},\mathfrak{P})$	Esnek Topolojik Uzay
$\tilde{\pi}$	Skaler Değerli Parametrik Fonksiyon
$\mathbb{R}$	Reel Sayılar Cümlesi

TEŞEKKÜR

Bu tez çalışmasında değerli bilgi ve birikimini benimle paylaşan, desteklerini esirgemeyen ve büyük bir ilgiyle bana faydalı olmak için ellerinden gelenin fazlasını sunan kendilerini tanımaktan ve öğrencisi olmaktan büyük onur duyduğum sayın hocalarım Prof. Dr. Ali MUTLU ve Dr. Öğr. Üyesi Simge ÖZTUNÇ'a teşekkürü borç biliyor ve şükranlarımı sunuyorum. Ayrıca çalışmam boyunca her anlamda bana destek olan ve bu günlere gelmemde büyük emeği olan aileme, tüm dostlarıma sonsuz teşekkürlerimi sunarım.

Watson LUNGU
Manisa, 2023

ÖZET

Yüksek Lisans Tezi

ESNEK METRİK UZAYLAR ÜZERİNDE ESNEK KOMPAKT KÜMELERİ İÇİN EN YAKIN NOKTASI

Watson LUNGU

Manisa Celal Bayar Üniversitesi
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Danışman: Prof. Dr. Ali MUTLU

Bu tez çalışmasında esnek metrik uzaylar üzerinde esnek kompakt kümeler için farklı tip büzülemeleri incelenmiştir. Esnek kompakt kümelerin göz önüne alınmasıyla esnek metrik uzaylarda en yakın nokta elde edilmiştir. Bu takdirde esnek metrik uzaylarda bazı esnek sabit nokta teoremleri ispatlanmıştır.

Anahtar Kelimeler: Esnek Küme, Esnek Metrik Uzay, En Yakın Nokta.

2023, 43 sayfa

ABSTRACT

M. Sc. Thesis

BEST PROXIMITY POINT OF SOFT COMPACT SETS ON SOFT METRIC SPACES

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In this thesis, different types of contractions for soft compact sets on soft metric spaces are investigated. In soft metric spaces, the best proximity point is obtained by considering soft compact sets. In this case, some soft fixed point theorems have been proved in soft metric spaces.

Keywords: Soft Set, Soft Metric, Best Proximity Point

2023, 43 pages

1. GİRİŞ

En yakın nokta ve bir dönüşümün sabit noktası matematiğin ilgi çekici konuları arasındadır. Son yıllarda araştırmacılar tarafından pek çok önemli sonuç elde edilmiştir. Diğer taraftan bir çok matematikçi tarafından çalışılan esnek küme teorisi matematiksel problemleri çözmek için teorik ve uygulamalı olarak alternatif bir araç oluşturur. Bu konunun geliştirilmesiyle yeni bir araştırma alanı ortaya çıkmıştır

Son yıllarda Molodtsov **([36])** esnek küme kavramını tanımladı ve Maji **([34])** esnek küme teorisinin bazı özellikleri üzerine çalıştı. Böylece araştırmacılar tarafından birçok önemli sonuçlar elde edildi ve esnek küme teorisi matematiksel problemleri çözmek için teorik ve uygulamalı açıdan alternatif bir araç haline geldi. En önemli esnek topolojik özelliklerden biri olan esnek kompaktlık Zorlutuna ve arkadaşları tarafından ortaya atıldı. Esnek metriğin 2013 yılında Das ve Samanta **([15-17])** tarafından tanımlanmasıyla ve 2017 yılında Hosseinzadeh **([27])** tarafından geliştirilmesiyle yeni bir araştırma alanı ortaya çıkmıştır. Diğer taraftan bir dönüşümün sabit noktası ve en yakın nokta matematik ve ilgili alanlarda ilgi çekici konular arasındadır. Son yıllarda farklı tip esnek büzülmeler tarif edilmiştir ve bu büzülmelerin kullanılmasıyla çeşitli türden esnek metrik uzaylarda çok önemli teoremler ve sonuçlar verilmiştir.

Bu çalışmanın amacı esnek metrik uzaylarda esnek kompakt kümeler için farklı tip büzülemeleri incelemektir. Esnek kompakt kümelerin göz önüne alınmasıyla esnek metrik uzaylarda bazı yakın noktalar elde edilmiştir. Bu takdirde esnek metrik uzaylarda bazı esnek sabit nokta teoremleri ispatlanmıştır.

2. GENEL BİLGİLER

Bu bölüm esnek cümle teorisi ve sabit nokta teorisi ile ilgili temel kavramlardan söz edilmiştir.

2.1. Esnek Küme Teorisi

Tanım 2.1.1. [27] $\mathfrak{K}$ evrensel bir cümle ve $\mathfrak{P}$ parametrelerin bir cümlesi olsun. $\mathfrak{P}(\mathfrak{K})$, $\mathfrak{K}$ 'in kuvvet cümlesi ve $\mathfrak{R} \subseteq \mathfrak{P}$ olmak üzere;

$$\mathfrak{F} : \mathfrak{R} \to \mathfrak{P}(\mathfrak{K}),$$

ise $(\mathfrak{F}, \mathfrak{R})$ çifti $\mathfrak{K}$ üzerinde bir esnek cümle olarak adlandırılır.

Herhangi bir $(\mathfrak{F}, \mathfrak{R})$ esnek cümlesi, her $\mathfrak{r} \in \mathfrak{R}$ için $\mathfrak{F}(\mathfrak{r}) \neq \varnothing$ ve her $\mathfrak{r} \in \mathfrak{P} \setminus \mathfrak{R}$ için $\mathfrak{F}(\mathfrak{r}) = \varnothing$ olmak üzere $(\mathfrak{F}, \mathfrak{R})$ şeklinde bir esnek cümleye genişletilebilir.

Tanım 2.1.2 [34] Bir esnek cümlesi, $(\mathfrak{F}, \mathfrak{R})$ esnek cümlesinin parametreli alt cümleler ailesidir. $\mathfrak{e} \in \mathfrak{F}$ için, $\mathfrak{F}(\mathfrak{e})$, $(\mathfrak{F}, \mathfrak{R})$ esnek cümlenin $\mathfrak{e}-$yaklaşım elemanların cümlesi olarak adlandırılır.

Örnek 2.1.3 [34] $(\mathfrak{F}, \mathfrak{R})$ esnek cümlesi Mrs.A nın giyeceği eteklerini parametrelere göre çekiciliğini tanımlansın. $\mathfrak{K} = \{\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3, \mathfrak{h}_4\}$ ve

$\mathfrak{P} = \{\mathfrak{p}_1 = ucuz, \mathfrak{p}_2 = pahalı, \mathfrak{p}_{36} = renkli\}$, bu eteklerinin özellikleri gösteren parametrelerin bir cümlesi olsun. Esnek cümle tanımlanacak olursa

$$\mathfrak{F}(\mathfrak{e}_1) = \{\mathfrak{h}_1, \mathfrak{h}_2\},\ \mathfrak{F}(\mathfrak{e}_2) = \{\mathfrak{h}_3, \mathfrak{h}_4\},\ \mathfrak{F}(\mathfrak{h}_3) = \{\mathfrak{h}_1, \mathfrak{h}_3, \mathfrak{h}_4\}$$

o halde $\{\mathfrak{F}(\mathfrak{e}_i) \mid\ i = 1,2,3\}$ $P(\mathfrak{X})$ 'in ailesi bir $(\mathfrak{F}, \mathfrak{R})$ esnek cümledir.

Tanım 2.1.4. [34] $\mathfrak{K}$ üzerinde iki $(\mathfrak{F}, \mathfrak{R})$ ve $(\mathfrak{L}, \mathfrak{Q})$ esnek cümlesinin kesişimi $(\mathfrak{H}, \mathfrak{C})$ esnek cümlesidir. $\mathfrak{C} = \mathfrak{R} \cap \mathfrak{Q}$ ve $\forall \mathfrak{c} \in \mathfrak{C}$ olmak üzere $\mathfrak{H}(\mathfrak{c}) = \mathfrak{F}(\mathfrak{c}) \cap \mathfrak{Q}(\mathfrak{c})$. Bu kesişim $(\mathfrak{F}, \mathfrak{R}) \tilde{\cap} (\mathfrak{L}, \mathfrak{Q}) = (\mathfrak{H}, \mathfrak{C})$ ile gösterilir.

Tanım 2.1.5. [34] $\mathfrak{K}$ üzerinde iki $(\mathfrak{F}, \mathfrak{R})$ ve $(\mathfrak{L}, \mathfrak{Q})$ esnek cümlesinin birleşimi yine

bir esnek cümledir. $\mathfrak{C} = \mathfrak{R} \cup \mathfrak{Q}$ ve $\forall \mathfrak{c} \in \mathfrak{C}$ için,

$$\mathfrak{H}(\varsigma)=\begin{cases}\mathfrak{F}(\mathfrak{e}), & \mathfrak{e}\in\mathfrak{R}-\mathfrak{Q}\\ \mathfrak{L}(\mathfrak{e}), & \mathfrak{e}\in\mathfrak{Q}-\mathfrak{R}\\ \mathfrak{F}(\mathfrak{e})\cup\mathfrak{L}(\mathfrak{e}), & \mathfrak{e}\in\mathfrak{R}\cap\mathfrak{Q}.\end{cases}$$

Bu bağıntı $(\mathfrak{F},\mathfrak{R})\breve{\cup}(\mathfrak{L},\mathfrak{Q})=(\mathfrak{H},\mathfrak{C})$ ile ifade edilir.

Tanım 2.1.6. [34] $\mathfrak{K}$ üzerindeki bir $(\mathfrak{F},\mathfrak{R})$ esnek cümlesi $\forall\mathfrak{r}\in\mathfrak{R}$ için $\mathfrak{F}(\mathfrak{r})=\varnothing$ ise bir boş (null) esnek cümle olarak tanımlanır ve Φ ile gösterilir.

Tanım 2.1.7. [34] $\mathfrak{K}$ üzerindeki $\tilde{\mathfrak{K}}$ mutlak esnek cümlesi $\forall\mathfrak{r}\in\mathfrak{R}$ için $\mathfrak{F}(\mathfrak{r})=\mathfrak{K}$ olacak biçimde ifade edilen bir $(\mathfrak{F},\mathfrak{R})$ esnek cümlesidir.

Tanım 2.1.8. [46] $\mathfrak{T}$, $\mathfrak{K}$ üzerinde esnek cümlelerin bir koleksiyonu olsun. Bu durumda $\mathfrak{T}$ aşağıdaki aksiyomları sağlıyorsa $\mathfrak{K}$ üzerinde bir esnek topoloji olarak adlandırılır.

i) $\Phi,\mathfrak{K}=(\mathfrak{F},\mathfrak{K})\in\mathfrak{T}$,

ii) $\mathfrak{T}$'daki herhangi (keyfi) sayıda esnek cümlelerin birleşimi $\mathfrak{T}$ bulunur.

iii) $\mathfrak{T}$' daki herhangi iki esnek cümlenin kesişimi yine $\mathfrak{T}$ bulunur.

$(\mathfrak{K},\mathfrak{T},\mathfrak{P})$ üçlüsüne $\mathfrak{K}$ üzerinde bir esnek topolojik uzay denir.

Tanım 2.1.9. [52] $(\mathfrak{K},\mathfrak{T},\mathfrak{P})$ ve $(\mathfrak{K}',\mathfrak{T}',\mathfrak{P})$ iki esnek topolojik uzay olsun. $f:(\mathfrak{K},\mathfrak{T},\mathfrak{P})\to(\mathfrak{K}',\mathfrak{T}',\mathfrak{P})$ bir dönüşüm olsun. $(f(\mathfrak{f}_p),\mathfrak{P})$' nin her bir $(\mathfrak{H},\mathfrak{P})$ esnek komşuluğu için $f((\mathfrak{F},\mathfrak{P}))\subset(\mathfrak{H},\mathfrak{P})$ olacak şekilde $(\mathfrak{f}_p,\mathfrak{P})$'nin bir $(\mathfrak{F},\mathfrak{P})$ esnek komşuluğu mevcutsa bu takdirde f'e $(\mathfrak{f}_p,\mathfrak{P})$'de esnek sürekli dönüşüm denir.

Lemma 2.1.10. [52] $(\mathfrak{K},\mathfrak{T},\mathfrak{P})$ esnek topolojik uzay olsun. $\acute{\imath}_{\mathfrak{K}}:\mathfrak{K}\to\mathfrak{K}$ birim dönüşümü esnek sürekli olmak zorunda değildir.

İspat: $\mathfrak{K}=\{\mathfrak{f}_1,\mathfrak{f}_2,\mathfrak{f}_3\}$, $\mathfrak{P}=\{\mathfrak{p}_1,\mathfrak{p}_2\}$ ve $\mathfrak{T}=\{\varnothing,\mathfrak{K},(\mathfrak{F}_1,\mathfrak{P}),(\mathfrak{F}_2,\mathfrak{P}),(\mathfrak{F}_3,\mathfrak{P}),(\mathfrak{F}_4,\mathfrak{P})\}$ ve $\mathfrak{T}'=\{\varnothing,\mathfrak{K},(\mathfrak{L}_1,\mathfrak{P}),(\mathfrak{L}_2,\mathfrak{P}),(\mathfrak{L}_3,\mathfrak{L}),(\mathfrak{L}_4,\mathfrak{P})\}$, $\mathfrak{K}$ üzerinde iki esnek topolojik uzay ve $(\mathfrak{F}_1,\mathfrak{P}),(\mathfrak{F}_2,\mathfrak{P}),(\mathfrak{F}_3,\mathfrak{P}),(\mathfrak{F}_4,\mathfrak{P}),(\mathfrak{L}_1,\mathfrak{P}),(\mathfrak{L}_2,\mathfrak{P}),(\mathfrak{L}_3,\mathfrak{P}),(\mathfrak{L}_4,\mathfrak{P})$, $\mathfrak{K}$ üzerinde esnek açık cümleler olmak üzere $\mathfrak{F}_1(\mathfrak{p}_1)=\{\mathfrak{f}_2\},\mathfrak{F}_1(\mathfrak{p}_2)=\{\mathfrak{f}_1\}$,

$\mathfrak{F}_2(\mathfrak{p}_1)=\{\mathfrak{f}_2,\mathfrak{f}_3\},\ \mathfrak{F}_2(\mathfrak{p}_1)=\{\mathfrak{f}_1\},\mathfrak{F}_2(\mathfrak{p}_2)=\{\mathfrak{f}_1,\mathfrak{f}_2\},\mathfrak{F}_3(\mathfrak{p}_1)=\{\mathfrak{f}_3\},$

$\mathfrak{F}_3(\mathfrak{p}_2)=\{\mathfrak{f}_1,\mathfrak{f}_2\},\ \mathfrak{F}_4(\mathfrak{p}_1)=\varnothing,\ \mathfrak{F}_2(\mathfrak{p}_2)=\{\mathfrak{f}_1\}$ ve

$\mathfrak{L}_1(\mathfrak{p}_1)=\{\mathfrak{f}_2\},\ \mathfrak{L}_1(\mathfrak{p}_2)=\{\mathfrak{f}_1\},\ \mathfrak{L}_2(\mathfrak{p}_1)=\{\mathfrak{f}_2,\mathfrak{f}_3\},\ \mathfrak{L}_2(\mathfrak{p}_2)=\{\mathfrak{f}_1,\mathfrak{f}_2\},\ \mathfrak{L}_3(\mathfrak{p}_1)=\{\mathfrak{f}_1,\mathfrak{f}_2\},$

$\mathfrak{L}_3(\mathfrak{p}_2)=\mathfrak{K},\ \mathfrak{L}_4(\mathfrak{p}_1)=\{\mathfrak{f}_2\},\mathfrak{L}_4(\mathfrak{p}_2)=\{\mathfrak{f}_1,\mathfrak{f}_2\}$ olacak biçimde verilsin. $f^{-1}((\mathfrak{L}_3,\mathfrak{P}))=\{(\mathfrak{p}_1,\{\mathfrak{f}_1,\mathfrak{f}_2\}),(\mathfrak{p}_2,\mathfrak{K})\}\notin(\mathfrak{K},\mathfrak{T},\mathfrak{P})$ ve

$f^{-1}((\mathfrak{L}_4,\mathfrak{P}))=\{(\mathfrak{p}_1,\{\mathfrak{f}_2\}),(\mathfrak{p}_2,\{\mathfrak{f}_1,\mathfrak{f}_2\})\}\notin(\mathfrak{K},\mathfrak{T},\mathfrak{P})$. Bundan dolayı $f=\mathfrak{i}_{\mathfrak{K}}:\mathfrak{K}\to\mathfrak{K}$ dönüşümü esnek sürekli değildir.

Tanım 2.1.11. [35] $\mathfrak{R}\subseteq\mathfrak{P}$ parametrelerin bir kümesi olsun. Eğer $r\in\mathbb{R}$ ve $\mathfrak{r}\in\mathfrak{R}$ ise $(\mathfrak{r},r)$ sıralı ikilisini esnek parametrik skaler olduğunu söyleriz. $r\geq 0$ ise $(\mathfrak{r},r)$ esnek parametrik skaleri negatif olmayan olarak adlandırılır. $(\mathfrak{r},r)$ ve $(\mathfrak{r}',r')$ iki esnek parametrik skalerler olsun. $r\geq r'$ ise $(\mathfrak{r},r)$'nin $(\mathfrak{r}',r')$'den küçük olmadığını söyleriz ve $(\mathfrak{r},r)\succeq(\mathfrak{r}',r')$ şeklinde yazarız.

Tanım 2.1.12. [35] $\mathfrak{R}\subseteq\mathfrak{P}$ parametrelerin bir cümlesi olsun. $(\mathfrak{r},r)$ ve $(\mathfrak{r}',r')$ iki esnek parametrik skaler olsun. Bu durumda esnek parametrik skalerler arasındaki toplamayı ve esnek parametrik skalerler üzerindeki skalerler çarpımını aşağıdaki gibi verilir.

$(\mathfrak{r},r)\dot{+}(\mathfrak{r}',r')=(\{\mathfrak{r},\mathfrak{r}'\},r+r')$ ve her $\lambda\in\mathbb{R}$ için $\lambda(a,r)=(\mathfrak{r},\lambda r)$.

Tanım 2.1.13. [35] $(\mathfrak{F},\mathfrak{R})$, $\mathfrak{K}$ üzerinde bir esnek cümle olsun.

$f(\mathfrak{F},\mathfrak{R})=(f_1,f_2)(\mathfrak{R},\mathfrak{F}(\mathfrak{R}))$ olacak şekilde $f_1:\mathfrak{R}\to\mathfrak{R}$ ve $f_2:\mathfrak{F}(\mathfrak{R})\to\mathbb{R}$

fonksiyonları varsa $(\mathfrak{F},\mathfrak{R})$ üzerindeki bir f fonksiyonu parametrik skaler değerli bir fonksiyon olarak adlandırılır. Benzer şekilde yukarıdaki tanımı $f_1:\mathfrak{R}\times\mathfrak{R}\to\mathfrak{R}$ ve $f_2:\mathfrak{F}(\mathfrak{R})\times\mathfrak{F}(\mathfrak{R})\to\mathbb{R}$ olmak üzere

$$f(\mathfrak{R}\times\mathfrak{R},\mathfrak{F}(\mathfrak{R})\times\mathfrak{F}(\mathfrak{R}))=(f_1,f_2)(\mathfrak{R}\times\mathfrak{R},\mathfrak{F}(\mathfrak{R})\times\mathfrak{F}(\mathfrak{R}))$$

ile $f:(\mathfrak{F},\mathfrak{R})\times(\mathfrak{F},\mathfrak{R})\to(\mathfrak{R},\mathbb{R})$ şeklinde genişletilir.

2.2. Metrik Uzaylar ve Esnek Metrik Uzaylar

Tanım 2.2.1. [37] $\mathcal{K}$ boştan farklı bir cümle olmak üzere $\mathfrak{h}: \mathcal{K} \times \mathcal{K} \to \mathbb{R}^+$ fonksiyonu

M(1) $\quad \forall \mathfrak{x}, \mathfrak{g} \in \mathcal{X},\ (\mathfrak{x}, \mathfrak{g}) \geq 0$

M(2) $\quad \forall \mathfrak{x}, \mathfrak{g} \in \mathcal{K},\ \mathfrak{h}(\mathfrak{x}, \mathfrak{g}) = 0 \Leftrightarrow \mathfrak{x} = \mathfrak{g}$

M(3) $\quad \forall \mathfrak{x}, \mathfrak{g} \in \mathcal{K},\ \mathfrak{h}(\mathfrak{x}, \mathfrak{g}) = \mathfrak{h}(\mathfrak{x}, \mathfrak{g})$

M(4) $\quad \forall \mathfrak{x}, \mathfrak{g}, \mathfrak{y} \in \mathcal{X},\ \mathfrak{h}(\mathfrak{x}, \mathfrak{g}) \leq \mathfrak{h}(\mathfrak{x}, \mathfrak{g}) + \mathfrak{h}(\mathfrak{g}, \mathfrak{y})$

özelliklerini sağlıyorsa $\mathfrak{h}$ ye $\mathcal{K}$ üzerinde bir metrik ve $(\mathcal{K}, \mathfrak{h})$ ikilisine de metrik uzay denir.

Tanım 2.2.2. [19] $\mathcal{K} \neq \varnothing$ ve $\mathfrak{x} \in \mathcal{K}$ olsun. $f: \mathcal{K} \to \mathcal{K}$ fonksiyonu için $f(\mathfrak{x}) = \mathfrak{x}$ ise $\mathfrak{x} \in \mathcal{K}$ 'e f 'in sabit noktası denir.

Tanım 2.2.5. [19] $(\mathcal{K}, \mathfrak{h})$ metrik uzay ve $\mathfrak{T}: \mathcal{K} \to \mathcal{K}$ herhangi bir dönüşüm olsun. Eğer her $\mathfrak{x}, \mathfrak{g} \in \mathcal{K}$ için

$$\mathfrak{h}(\mathfrak{T}\mathfrak{x}, \mathfrak{T}\mathfrak{g}) \leq \lambda\, \mathfrak{h}(\mathfrak{x}, \mathfrak{g})$$

şartını sağlayan bir $\lambda \geq 0$ sabiti varsa $\mathfrak{T}$ 'ye Lipschitz dönüşümü (Lipschitz mapping) denir. Burada $0 \leq \lambda < 1$ ise $\mathfrak{T}$ 'ye büzülme veya daralma (contraction) dönüşümü ve $\lambda = 1$ ise $\mathfrak{T}$ 'ye genişlemeyen (nonexpansive) dönüşüm denir. Eğer her $\mathfrak{x}, \mathfrak{g} \in \mathcal{K}$ için $(\mathfrak{x} \neq \mathfrak{g})$

$$\mathfrak{h}(\mathfrak{T}\mathfrak{x}, \mathfrak{T}\mathfrak{g}) \leq \lambda\, \mathfrak{h}(\mathfrak{x}, \mathfrak{g})$$

şartı sağlanıyorsa $\mathfrak{T}$ 'ye büzülebilir (contractive) dönüşüm denir. Bu durumda aşağıdaki gerektirmenin doğruluğu kolayca görülebilir.

Büzülme $\Rightarrow$ Büzülebilir $\Rightarrow$ Genişlemeyen $\Rightarrow$ Lipschitz.

Teorem 2.2.6. [11] $(\mathcal{K}, \mathfrak{h})$ bir tam metrik uzay ve $\mathfrak{T}: \mathcal{K} \to \mathcal{K}$ bir dönüşüm olmak üzere her $\mathfrak{x}, \mathfrak{g} \in \mathcal{K}$ için

$$\mathfrak{h}(\mathfrak{T}\mathfrak{x}, \mathfrak{T}\mathfrak{g}) \leq \lambda\, \mathfrak{h}(\mathfrak{x}, \mathfrak{g})$$

olacak şekilde bir $\lambda \in (0,1)$ sabiti bulunabiliyorsa $\mathfrak{T}$ 'nin $\mathfrak{K}$ 'de bir tek sabit noktası vardır.

Tanım 2.2.2. $\pi : \mathfrak{R} \times \mathfrak{R} \to \mathfrak{R}$ bir parametrik skaler değerli fonksiyon olsun. Bu takdirde $(\pi(\mathfrak{r}, \mathfrak{r}'), 0)$ esnek parametrik skaleri $(\pi(\mathfrak{r}, \mathfrak{r}'), 0) \in \mathfrak{R},\ 0 \in \mathbb{R}$ olmak üzere esnek anlamda 0 (sıfır) elemanını gösterir.

Tanım 2.2.3. [35] $(\mathfrak{F}, \mathfrak{R}),\ \mathfrak{K}$ üzerinde bir esnek küme olsun. $\tilde{\pi} : \mathfrak{R} \times \mathfrak{R} \to \mathfrak{R}$ bir parametrik değerli bir fonksiyon olsun. $\tilde{\mathfrak{D}}$ aşağıdaki şartları sağlıyorsa

$\tilde{\mathfrak{D}} : (\mathfrak{F}, \mathfrak{R}) \times (\mathfrak{F}, \mathfrak{R}) \to (\mathfrak{R}, \mathbb{R}^+ \cup \{0\})$ parametrik skaler değerli fonksiyonunun $(\mathfrak{F}, \mathfrak{R}),\ \mathfrak{K}$ üzerinde bir esnek metrik olduğunu söylenir.

1. $\tilde{\mathfrak{D}}((\mathfrak{r}, \mathfrak{F}(\mathfrak{r})), (\mathfrak{r}', \mathfrak{F}(\mathfrak{r}'))) \succeq (\tilde{\pi}(\mathfrak{r}, \mathfrak{r}'), 0),\ \mathfrak{r} = \mathfrak{r}'$ iken eşitlik sağlanır.
2. Her $\mathfrak{r}, \mathfrak{r}' \in \mathfrak{R}$ için $\tilde{\mathfrak{D}}((\mathfrak{r}, \mathfrak{F}(\mathfrak{r})), (\mathfrak{r}', \mathfrak{F}(\mathfrak{r}'))) = \tilde{\mathfrak{D}}((\mathfrak{r}', \mathfrak{F}(\mathfrak{r}')), (\mathfrak{r}, \mathfrak{F}(\mathfrak{r})))$
3. Her $\mathfrak{r}, \mathfrak{r}', \mathfrak{r}'' \in \mathfrak{R}$ için

$$\tilde{\mathfrak{D}}((\mathfrak{r}, \mathfrak{F}(\mathfrak{r})), (\mathfrak{r}'', \mathfrak{F}(\mathfrak{r}''))) \prec \tilde{\mathfrak{D}}((\mathfrak{r}, \mathfrak{F}(\mathfrak{r})), (\mathfrak{r}', \mathfrak{F}(\mathfrak{r}'))) \dot{+} \tilde{\mathfrak{D}}((\mathfrak{r}', \mathfrak{F}(\mathfrak{r}')), (\mathfrak{r}'', \mathfrak{F}(\mathfrak{r}'')))$$

$((\mathfrak{F}, \mathfrak{R}), \tilde{\mathfrak{D}})$ ikilisine $\mathfrak{K}$ üzerinde bir esnek metrik uzay denir.

3.MATERYAL VE YÖNTEMLER

Bu bölümde esnek kompakt metrik uzay tanımı verilerek, esnek açık örtüsü, esnek sabit nokta, esnek dizi, esnek yakınsak dizi, esnek Cauchy dizisi tanımları verilmiştir.

3.1. Esnek Kompakt Metrik Uzaylar

Tanım 3.1 [51] $(\mathfrak{F},\mathfrak{R})$'da tanımlı esnek açıklardan oluşan bir $\{(\mathfrak{F}_i,\mathfrak{R})\mid \mathfrak{i}\in\mathfrak{I}\}$ ailesi için,

$$(\mathfrak{F},\mathfrak{R})\subseteq\bigcup_{\mathfrak{i}\in\mathfrak{I}}(\mathfrak{F}_i,\mathfrak{R})$$

ise $(\mathfrak{F}_\mathfrak{i},\mathfrak{R})$ ailesine $(\mathfrak{F},\mathfrak{R})$'nın bir esnek açık örtüsü denir. Ayrıca, $(\mathfrak{F}_\mathfrak{i},\mathfrak{R})$'nın sonlu bir altailesi aynı zamanda $(\mathfrak{F},\mathfrak{R})$'nın bir esnek açık örtüsü ise, bu alt örtüye $(\mathfrak{F}_\mathfrak{i},\mathfrak{R})$'nin sonlu bir alt örtüsü denir.

Tanım 3.2 [51] $(\mathfrak{F},\mathfrak{R})$ uzayının her açık esnek örtüsü uzayını örten sonlu bir altörtüye sahipse, esnek uzayına esnek kompakttır denir.

Tanım 3.3 [27] $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{O}})$ bir esnek kompakt uzay ve

$$f:((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{O}})\to((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{O}})$$

bir esnek dönüşüm olsun. $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\subseteq(\mathfrak{F},\mathfrak{R})$, f'nin esnek bir sabit noktası öyleki

$$f((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))=(\mathfrak{a},\mathfrak{F}(\mathfrak{a})).$$

Tanım 3.4. $(\mathfrak{F},\mathfrak{R})$, $\mathfrak{K}$ üzerinde bir esnek kompakt cümle olsun. $(\mathfrak{F},\mathfrak{R})$'daki bir esnek dizi $n\in\mathbb{N}$ için $(\mathfrak{F}_n,\mathfrak{R})$, $(\mathfrak{F},\mathfrak{R})$'nın bir esnek alt cümlesi olacak şekilde $f(n)=(\mathfrak{F}_n,\mathfrak{R})$ ile tanımlı bir $f:\mathbb{N}\to(\mathfrak{F},\mathfrak{R})$ fonksiyonudur ve $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$ ile ifade edilir.

Tanım 3.5. $(\mathfrak{F},\mathfrak{R})$, $\mathfrak{K}$ üzerinde bir esnek kompakt cümle olsun. $\tilde{\mathfrak{D}}$, $(\mathfrak{F},\mathfrak{R})$ üzerinde bir esnek metrik $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty},(\mathfrak{F},\mathfrak{R})$'da bir esnek dizi ve $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))\in(\mathfrak{F},\mathfrak{R})$ olsun. Bu takdirde her

ϵ pozitif sayısı için $n\geq n_0(\epsilon)$ olmak üzere her n doğal sayısı için $\tilde{\mathfrak{D}}((\mathfrak{r},\mathfrak{F}_n(\mathfrak{r})),(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))\leq(\tilde{\pi}(\mathfrak{r},\mathfrak{ß}),\epsilon)$ olacak şekilde bir $n_0(\epsilon)$ doğal sayısı mevcut ise $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$ dizisi $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))$ 'e yakınsaktır denir.

Tanım 3.6. $(\mathfrak{F},\mathfrak{R})$, $\mathfrak{X}$ üzerinde bir esnek kompakt cümle olsun. $\tilde{\mathfrak{D}}$, $(\mathfrak{F},\mathfrak{R})$ üzerinde bir esnek metrik, $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$, $(\mathfrak{F},\mathfrak{R})$ 'da bir esnek dizi olsun. Bu durumda her ϵ pozitif sayısı için $n,m\geq n_0(\epsilon)$ olmak üzere her n,m doğal sayısı için $\tilde{\mathfrak{D}}((\mathfrak{r},\mathfrak{F}_n(\mathfrak{r})),(\mathfrak{r},\mathfrak{F}_m(\mathfrak{r})))\leq(\tilde{\pi}(\mathfrak{r},\mathfrak{r}),\epsilon)$ olacak şekilde bir $n_0(\epsilon)$ doğal sayısı mevcut ise $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$ dizisine bir Cauchy dizisi denir.

Önerme 3.7. $(\mathfrak{F},\mathfrak{R})$, $\mathfrak{K}$ üzerinde bir esnek kompakt cümle olsun. $\tilde{\mathfrak{D}}$, $(\mathfrak{F},\mathfrak{R})$ üzerinde bir esnek metrik, $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$, $(\mathfrak{F},\mathfrak{R})$ 'da bir esnek Cauchy dizisi olsun. $\{(\mathfrak{F}_n,\mathfrak{R})\}_{n=1}^{\infty}$, $(\mathfrak{F},\mathfrak{R})$ 'da esnek yakınsak ise bu takdirde $(\mathfrak{F},\mathfrak{R})$'nın bir tek elamanına yakınsar.

3.2.Eilenberg-Montgomery Sabit Nokta Teorisi

Doğrusal olmayan fonksiyel analizin birçok yönünde temel bir rol oynayan 1922'de kanıtlanan Banach büzülme dönüşümü metrik uzaylarda sabir nokta teorisi oldukça önemlidir. İlke, $\mathfrak{T}(\mathfrak{ß})=\mathfrak{ß}$ operatör denkleminin varlığını ve tekliğini oluşturmak için, 1890'da Picard, ardışık yaklaşımların kullanılmasından elde edilmiştir.

Ayrıca, son zamanlarında, bilinen sabit nokta teoremlerinin çeşitli sürümleri oluşturulmuştur. Örnek olarak, Von Neumann'ın oyun teorisi üzerine çalışması çok değerli dönüşümler için Bohnenblust-Karlin sabit nokta teoreminin bir genişletmesini Eilenberg ve Montgomery inşaa etti.

Öncelikle, metrik uzaylarda geri çekilme hatırlatalım.

$\mathcal{K}$ bir metrik uzay ve $\mathcal{R} \subset \mathcal{K}$ olsun. Eğer $r : \mathcal{K} \to \mathcal{R}$ olacak şekilde $r / \mathcal{K} = id_{\mathcal{K}}$ ise, r fonksiyonu geri çekilme denir. Eğer $\mathcal{U}$, $\mathcal{K}$'de $\mathcal{R}$'nın komşuluğu ve $r : \mathcal{U} \to \mathcal{R}$ olacak şekilde $r / \mathcal{K} = id_{\mathcal{K}}$ ise, r bir komşuluğu geri çekilmesi adlandılır. Bu takdirde $\mathcal{K}$'de $\mathcal{R}$'nin gömme $i : \mathcal{R} \to \mathcal{K}$ olarak gösterilir ve $r / \mathcal{K} = roi$.

Tanım 3.1 (Esnek sabit nokta uzay);

$\mathcal{T}$ bir sürekli dönüşüm olsun. Eğer $(\mathcal{F}, \mathcal{R})$'nın her $\mathcal{T}$ uzayının sürekli öz dönüşümü bir esnek sabit noktaya sahip ise, yani

$$\mathcal{T}(\mathfrak{h}_0, \mathcal{F}(\mathfrak{h}_0)) = \mathcal{T}(\mathfrak{h}_0, \mathcal{I}(\mathfrak{h}_0)) = (\mathfrak{h}_0, \mathcal{F}(\mathfrak{h}_0))$$

ise $(\mathcal{F}, \mathcal{R})$ ya esnek sabit uzaydır denir.

Teorem 3.2 [22] (Eilenberg-Montgomery Sabit Nokta Teorisi)

Boş olmayan bir $((\mathcal{F}, \mathcal{R}), \tilde{\Theta})$ esnek metrik uzayı, bir esnek kompakt devirli mutlak komşuluğu geri çekilim ise, o zaman esnek sabit nokta özelliğine sahiptir.

Teorem 3.3 $((\mathcal{F}, \mathcal{R}), \tilde{\Theta})$ bir esnek kompakt metrik uzay ve $\mathcal{T}$, $(\mathcal{F}, \mathcal{R})$'de bir esnek dönüşüm olsun. $(\mathfrak{h}, \mathcal{F}(\mathfrak{h})) \neq (\mathfrak{g}, \mathcal{F}(\mathfrak{g}))$ olmak üzere her $(\mathfrak{h}, \mathcal{F}(\mathfrak{h})), (\mathfrak{g}, \mathcal{F}(\mathfrak{g})) \in (\mathcal{F}, \mathcal{R})$ için

$$\tilde{\Theta}(\mathcal{T}((\mathfrak{h}, \mathcal{F}(\mathfrak{h}))), \mathcal{T}((\mathfrak{g}, \mathcal{F}(\mathfrak{g})))) \prec \tilde{\Theta}((\mathfrak{h}, \mathcal{F}(\mathfrak{h})), (\mathfrak{g}, \mathcal{F}(\mathfrak{g})))$$

olsun. O halde $\mathcal{T}$ bir tek esnek sabit noktaya sahiptir.

Teorem 3.4 $((\mathcal{F}, \mathcal{R}), \tilde{\Theta})$ bir esnek kompakt metrik uzay ve $\mathcal{T}$, $(\mathcal{F}, \mathcal{R})$'da bir esnek dönüşüm olsun. Her $(\mathfrak{h}, \mathcal{F}(\mathfrak{h})), (\mathfrak{g}, \mathcal{F}(\mathfrak{g})) \in (\mathcal{F}, \mathcal{R})$ için

$$\frac{1}{2}\tilde{\Theta}((\mathfrak{h}, \mathcal{F}(\mathfrak{h})), \mathcal{T}(\mathfrak{h}, \mathcal{F}(\mathfrak{h}))) \prec \tilde{\Theta}((\mathfrak{h}, \mathcal{F}(\mathfrak{h})), (\mathfrak{g}, \mathcal{F}(\mathfrak{g})))$$

olsun. Buradan

$$\tilde{\Theta}(\mathcal{T}((\mathfrak{h}, \mathcal{F}(\mathfrak{h}))), \mathcal{T}((\mathfrak{g}, \mathcal{F}(\mathfrak{g})))) \prec \tilde{\Theta}((\mathfrak{h}, \mathcal{F}(\mathfrak{h})), (\mathfrak{g}, \mathcal{F}(\mathfrak{g})))$$

O zaman $\mathcal{T}$ bir tek esnek sabit noktaya sahiptir.

Teorem 3.5 $\mathcal{K}$, $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$ esnek kompakt metrik uzayından yine kendisine esnek sürekli dönüşüm olsun ve $(\mathfrak{f},\mathfrak{F}(\mathfrak{f}))\neq(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))$ olmak üzere her $(\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\in(\mathfrak{F},\mathfrak{R})$ için

$$\tilde{\mathfrak{D}}(\mathcal{K}^n(\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathcal{K}^n(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))\prec\max\{\tilde{\mathfrak{D}}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))),\tilde{\mathfrak{D}}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathcal{K}(\mathfrak{f},\mathfrak{F}(\mathfrak{f})))$$

$$\tilde{\mathfrak{D}}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathcal{K}(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))),\frac{1}{2}[\mathfrak{D}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathcal{K}(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))\dot{+}\tilde{\mathfrak{D}}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathcal{K}(\mathfrak{f},\mathfrak{F}(\mathfrak{f})))]\}\quad(3.1)$$

eşitsizliği sağlansın. O halde $\mathcal{K}$, $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$'de bir tek esnek sabit noktaya sahiptir.

İspat $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$ esnek kompakt metrik uzay ve

$$f((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathcal{K}(\mathfrak{f},\mathfrak{F}(\mathfrak{f})))=\tilde{\mathfrak{D}}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathcal{K}(\mathfrak{f},\mathfrak{F}(\mathfrak{f})))$$

f, $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$'de bir parametrik skaler değerli fonksiyonu olsun. $\mathcal{K}$ esnek sürekli olduğundan f de esnek süreklidir. O halde, $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$, f'nin maksimum ve minimum sahiptir öyleki

$$f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))=\min\{f((\mathfrak{f},\mathfrak{F}(\mathfrak{f})))\mid\ (\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})\}\qquad(3.2)$$

$(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))$, $\mathcal{K}$'nin bir esnek sabit nokta olduğunu göstereceğiz. $f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))=\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\ \mathcal{K}(\mathfrak{a},f(\mathfrak{a})))\succ(\pi(\mathfrak{a},\mathfrak{a}),0)$ olsun, n parametrik pozitif tamsayı değer olduğunda ve $n=n((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\ \mathcal{K}\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))$ için (3.1)'den

$$f(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))=\tilde{\mathfrak{D}}(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))=\mathfrak{F}(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}^n\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))$$

$$\prec\max\left\{\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\tilde{\mathfrak{D}}(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\right.$$

$$\frac{1}{2}\left[\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}^2(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\dot{+}(\pi(\mathfrak{a},\mathfrak{a}),0)\right]$$

$$\preceq\max\left\{f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),f(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),f(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\frac{1}{2}[f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))+f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))]\right\}$$

elde edilir. (3.2)'den dolayı

$$\max\{f((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))), f(\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))), \frac{1}{2}[f((\mathfrak{a},\mathfrak{Jf}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))) \dot{+} f(\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))))]\} = f((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))$$

$$f(\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))) \preceq f((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))$$

bulunur, ayrıca bu (3.2) ile çelişir. Böylece, $(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))$, $\mathcal{K}$'nin bir esnek sabit noktasıdır. Şimdi $(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))$'nun tek olduğunu gösterelim. $(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})) \neq (\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))$ başka esnek sabit noktası olduğunu kabul edelim. Bu durumda

$$\mathcal{K}((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) = (\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}')) \Rightarrow \mathcal{K}^2((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) = (\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))\ldots\mathcal{K}^n((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) = (\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))$$

$$\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) = \tilde{\mathfrak{D}}(\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))),\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))) = \tilde{\mathfrak{D}}(\mathcal{K}^n((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))),\mathcal{K}^n((\mathfrak{a}^n,\mathfrak{Jf}(\mathfrak{a}^n))))$$

$$\preceq \max\left\{\begin{array}{l}\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{v}'))),\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))),\tilde{\mathfrak{D}}((\mathfrak{a}',\mathfrak{I}(\mathfrak{a}')),\mathcal{K}(\mathfrak{a}',\mathfrak{I}(\mathfrak{a}'))),\\ \frac{1}{2}[\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),\mathcal{K}((\mathfrak{a}',\mathfrak{I}(\mathfrak{a}')))) \dot{+} \tilde{\mathfrak{D}}((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}')),\mathcal{K}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))))]\end{array}\right\}$$

$$\preceq \max\left\{\begin{array}{l}\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))),\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))),\tilde{\mathfrak{D}}((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}')),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))),\\ \frac{1}{2}[\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) \dot{+} \tilde{\mathfrak{D}}((\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}')),(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})))]\end{array}\right\}$$

$$\preceq \tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))).$$

Yukarıdaki eşitsizliği kullanarak

$$\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}'))) \prec \tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{Jf}(\mathfrak{a}')))$$

bulunur. Bu bir çelişkidir, O halde $\mathcal{K}$'nin esnek sabit naktası tektir.

Örnek 3.6 $\tilde{\mathfrak{D}}$, $\mathfrak{K} = [0,\infty)$, $\mathfrak{P} = Q^+$ de bir metrik ve $(\mathfrak{Jf},\mathfrak{P})$, $\mathfrak{K}$ üzerinde bir esnek küme olsun, öyleki her $(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})) \in (\mathfrak{Jf},\mathfrak{P})$ için $(\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})) = [0,\mathfrak{a}]$ olarak tanımlansın.

$$\tilde{\mathfrak{D}} : (\mathfrak{Jf},\mathfrak{P}) \times (\mathfrak{Jf},\mathfrak{P}) \to (\mathfrak{R}, \mathbb{R}^+ \cup 0)$$

$$\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a})),(\mathfrak{b},\mathfrak{Jf}(\mathfrak{b}))) = (\max\{\mathfrak{a},\mathfrak{b}\},\mathfrak{h}(\mathfrak{a},\mathfrak{b}))$$

esnek metrik olsun. $\mathfrak{T}$ dönüşümü aşağıdaki gibi tanımlayalım.

$$\mathfrak{T} : ((\mathfrak{Jf},\mathfrak{P}),\tilde{\mathfrak{D}}) \to ((\mathfrak{Jf},\mathfrak{P}),\tilde{\mathfrak{D}})$$

$$\mathfrak{T}((\mathfrak{a},\mathfrak{Jf}(\mathfrak{a}))) = \left(\frac{1}{2}\mathfrak{a},\mathfrak{Jf}\left(\frac{1}{2}\mathfrak{a}\right)\right) = \left[0,\frac{1}{2}\right]$$

Buradan

$$\tilde{\Phi}(\mathfrak{T}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathfrak{T}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})))=\tilde{\Phi}\left(\left(\frac{1}{2}\mathfrak{a},\mathfrak{F}\left(\frac{1}{2}\mathfrak{a}\right)\right),\left(\frac{1}{2}\mathfrak{b},\mathfrak{F}\left(\frac{1}{2}\mathfrak{b}\right)\right)\right)$$

$$=\left(\max\left\{\frac{1}{2}\mathfrak{a},\frac{1}{2}\mathfrak{b}\right\},\mathfrak{h}\left(\frac{1}{2}\mathfrak{a},\frac{1}{2}\mathfrak{b}\right)\right)$$

$$=\frac{1}{2}(\{\mathfrak{a},\mathfrak{b}\},\mathfrak{h}(\mathfrak{a},\mathfrak{b}))$$

$$=\frac{1}{2}\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b})))$$

Şimdi n parametrik pozitif tamsayı için

$$\tilde{\Phi}(\mathfrak{T}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathfrak{T}^n(\mathfrak{b},\mathfrak{F}(\mathfrak{b})))=\tilde{\Phi}\left(\left(\frac{1}{2^n}\mathfrak{a},\mathfrak{F}\left(\frac{1}{2^n}\mathfrak{a}\right)\right),\left(\frac{1}{2^n}\mathfrak{b},\mathfrak{F}\left(\frac{1}{2^n}\mathfrak{b}\right)\right)\right)$$

$$=\left(\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},\frac{1}{2^n}|\mathfrak{a}-\mathfrak{b}|\right)$$

$$=\max\left\{\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},\left|\frac{1}{2^n}\mathfrak{a}-\frac{1}{2^n}\mathfrak{b}\right|\right\}$$

$$=\left\{\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\}\mathfrak{b},\ \left|\mathfrak{T}^n(\mathfrak{a})-\mathfrak{T}^n(\mathfrak{b})\right|\right\}$$

$$\prec\left(\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},\max\left\{|\mathfrak{a}-\mathfrak{b}|,|\mathfrak{a}-\mathfrak{T}(\mathfrak{a})|,|\mathfrak{b}-\mathfrak{T}(\mathfrak{b})|,\frac{1}{2}[|\mathfrak{a}-\mathfrak{T}(\mathfrak{b})|\dot{+}|\mathfrak{b}-\mathfrak{T}(\mathfrak{a})|]\right\}\right)$$

$$\prec\max\left(\begin{array}{l}\left(\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},\ |\mathfrak{a}-\mathfrak{b}|\right),\ \left(\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},\ |\mathfrak{a}-\mathfrak{T}(\mathfrak{a})|\right),\\ \quad\left(\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{b}-\mathfrak{T}(\mathfrak{b})|\right),\\ \quad\frac{1}{2}\left[\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{a}-\mathfrak{T}(\mathfrak{b})|\dot{+}\max\left\{\frac{1}{2^n}\mathfrak{a},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{b}-\mathfrak{T}(\mathfrak{a})|\right]\end{array}\right)$$

$$=\max\left\{\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathfrak{T}(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\tilde{\Phi}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\right.$$

$$\frac{1}{2}[\tilde{\mathfrak{G}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathfrak{T}(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))+\tilde{\mathfrak{G}}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})),\mathfrak{T}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))]\Big\} \qquad (3.3)$$

$\mathfrak{T}$'nin (3.3) eşitsizliği sağladığı görülür. Böylece $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

4. ARAŞTIRMA BULGULARI

4.1 Esnek Yakın Nokta ve Esnek Kompakt Metrik Uzaylarda Esnek Sabit Nokta Teoremleri

$(\mathfrak{F},\mathfrak{R})$, $(\mathfrak{L},\mathfrak{Q})\subseteq\tilde{\mathfrak{K}}$ ve $\mathfrak{T}:(\mathfrak{F},\mathfrak{R})\to(\mathfrak{L},\mathfrak{Q})$ mutlaka sabit bir noktaya sahip olmayan bir kümenin kendisinden yine kendisine olmayan esnek dönüşümü göz önüne alalım. $\mathfrak{T}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))=(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))$ sabit nokta dekleminin çözüme sahip olmadığında, o zaman $\tilde{\mathfrak{G}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathfrak{T}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))$ minimum olacak şekilde yaklaşık bir $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))$ esnek noktasını belirlemeye çalışır.

Tanım 4.1 (En Yakın Nokta.) $(\mathfrak{F},\mathfrak{R})$ ve $(\mathfrak{L},\mathfrak{Q})$, $\tilde{\mathfrak{K}}$ esnek kompakt metrik uzayının

esnek alt kümeler olsun. Eğer $(\mathfrak{v},\mathfrak{F}(\mathfrak{v}))\in\tilde{\mathfrak{K}}$ için

$$\tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})),\mathcal{K}((\mathfrak{v},\mathfrak{F}(\mathfrak{v}))))=\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))$$

ve

$$\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))=\min\{\tilde{\mathfrak{G}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),(\mathfrak{g},\mathfrak{L}(\mathfrak{g}))):\mathfrak{ß}\in\mathfrak{R},\mathfrak{g}\in\mathfrak{L}\}$$

ise bir $(\mathfrak{v},\mathfrak{F}(\mathfrak{v}))$ noktasına en yakın noktasıdır denir.

Lemma 4.2 $(\mathfrak{F},\mathfrak{R})$ ve $(\mathfrak{L},\mathfrak{Q})$, $\tilde{\mathfrak{K}}$ esnek kompakt metrik uzayının boş olmayan esnek altcümleler olsun ve ω,k sabitler olmak üzere $0<\omega<1,\ k\in\mathbb{N}$ için

$$\mathfrak{T}_{n+k}\le\omega^k\mathfrak{T}_n+(1-\omega^k)\mathfrak{T}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q})) \qquad (4.1)$$

O halde

$n \to \infty$ iken $\mathfrak{T}_{n+k} \to \mathfrak{T}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))$.

Lemma 4.3 $(\mathfrak{F},\mathfrak{R})$ ve $(\mathfrak{L},\mathfrak{Q})$ esnek metrik uzayının boş olmayan esnek altcümleler olsun ve (4.1) sağlansın. O halde $(\mathfrak{F}_{2n+k},\mathfrak{R})$ ve $(\mathfrak{F}_{2n+k+1},\mathfrak{Q})$ diziler sınırlıdır.

Teorem 4.4 $(\mathfrak{F},\mathfrak{R})$ ve $(\mathfrak{L},\mathfrak{Q})$ $\tilde{\aleph}$ esnek metrik uzayının boş olmayan esnek altcümleleri olsun ve (4.1) sağlansın. $(\mathfrak{F},\mathfrak{R})$ veya $(\mathfrak{L},\mathfrak{Q})$ sınırlı esnek kompakt ise, bu durumda

$$\tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{I}(\mathfrak{v})),\mathfrak{T}((\mathfrak{v},\mathfrak{I}(\mathfrak{v})))) = \tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))$$

olacak şekilde bir $(\mathfrak{v},\mathfrak{F}(\mathfrak{v})) \in (\mathfrak{F},\mathfrak{R}) \cup (\mathfrak{L},\mathfrak{Q})$ vardır.

İspat μ çift sayı olsun. O halde $(\mathfrak{F}_{2n+k},\mathfrak{R}) \in (\mathfrak{F},\mathfrak{R})$. $(\mathfrak{F},\mathfrak{R})$ sınırlı kompakt olsun. $(\mathfrak{F}_{2n+k},\mathfrak{R})$ sınırlı esnek dizi olduğundan $(\mathfrak{F},\mathfrak{R})$ sınırlı esnek kompaktır. Dolayısıyla $(\mathfrak{F}_{2n+k},\mathfrak{R}),(\mathfrak{F},\mathfrak{R})$'da yakınsak bir esnek altdizisi sahiptir. Böylece $\eta \to \infty$ iken $(\mathfrak{F}_{2\eta+k},\mathfrak{R}) \to (\mathfrak{v},\mathfrak{F}(\mathfrak{v})) \in (\mathfrak{F},\mathfrak{R})$ olmak üzere $(\mathfrak{F}_{2n_\eta+k},\mathfrak{R}),(\mathfrak{F}_{2n+k},\mathfrak{R})$'nin esnek altdizisi olsun. $\mathcal{T}$ sürekli esnek dönüşüm olduğundan ve

$$\tilde{\mathfrak{G}}(\tilde{\mathfrak{v}}_{2n_1+k},\mathfrak{T}(\tilde{\mathfrak{v}}_{2n_1+k})) \to \tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q})).$$

O halde, $\tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})),\mathfrak{T}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})))) = \tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))$.

k tek sayı ve $(\mathfrak{L},\mathfrak{Q})$ sınırlı kompakt ise, ispat benzer şekilde yapılır.

Tanım 4.5 $\tilde{\aleph}$ bir esnek kompakt metrik uzay ve $\mathfrak{L}$ bir yönlendirilmiş grafığı olsun. $(\mathfrak{F},\mathfrak{R})$, $(\mathfrak{H},\mathfrak{Q})$, $\tilde{\aleph}$'nin boş olmayan esnek altcümleler olsun. Kendisinden yine kendine olmayan bir $\mathfrak{T}:(\mathfrak{F},\mathfrak{R}) \to (\mathfrak{H},\mathfrak{Q})$ dönüşümü

- eğer $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})),(\mathfrak{v},\mathfrak{F}(\mathfrak{v})),(\mathfrak{v}',\mathfrak{F}(\mathfrak{v}')) \in (\mathfrak{F},\mathfrak{R})$ için,

$$(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})) \in \mathfrak{P}(\mathfrak{L})$$

$$\tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})),\mathfrak{T}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))) = \tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{L}))$$

$$\tilde{\mathfrak{G}}((\mathfrak{v}',\mathfrak{I}(\mathfrak{v}')),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})))) = \tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{L},\mathfrak{Q}))$$

$$\Rightarrow \tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})),(\mathfrak{v}',\mathfrak{F}(\mathfrak{v}'))) \preceq \zeta[\tilde{\mathfrak{G}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{R})),(\mathfrak{v}',\mathfrak{F}(\mathfrak{v}'))) \dot{+} \tilde{\mathfrak{G}}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),(\mathfrak{v},\mathfrak{F}(\mathfrak{v})))]$$

olacak şekilde $\zeta \in [0,(1/2))$ varsa, bir $\mathfrak{L}-$proksimal Kannan dönüşümdür.

- Her $(\mathfrak{ƀ},\mathfrak{F}(\mathfrak{ƀ})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})),(\mathfrak{v},\mathfrak{F}(\mathfrak{v})),(\mathfrak{v}',\mathfrak{F}(\mathfrak{v}'))\in(\mathfrak{F},\mathfrak{R})$ için,

$$(\mathfrak{ƀ},\mathfrak{F}(\mathfrak{ƀ})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\in\mathfrak{P}(\mathfrak{L})$$

$$\tilde{\mathfrak{G}}((\mathfrak{v},\mathfrak{F}(\mathfrak{v})),\mathfrak{T}((\mathfrak{ƀ},\mathfrak{F}(\mathfrak{ƀ}))))=\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{L}))$$

$$\tilde{\mathfrak{G}}((\mathfrak{v}',\mathfrak{F}(\mathfrak{v}')),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))=\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

$$\Rightarrow(\mathfrak{v},\mathfrak{F}(\mathfrak{v})),(\mathfrak{v}',\mathfrak{F}(\mathfrak{v}'))\in\mathfrak{P}(\mathfrak{L})$$

ise proximal $\mathfrak{L}-$kenarı korumasıdır.

Toerem 4.6 $(\tilde{\mathfrak{K}},\tilde{\mathfrak{G}})$ bir esnek kompakt metrik uzay ve $\mathfrak{L}$ bir yönlendirilmiş grafığı olsun. $(\mathfrak{F},\mathfrak{R})$ ve $(\mathfrak{H},\mathfrak{Q})$, $\tilde{\mathfrak{K}}$'nin boş olmayan kapalı esnek altkümeler olsun. $(\mathfrak{F},\mathfrak{R}_0)\neq\varnothing$ olduğunu kabul edelim ve $\tilde{\mathfrak{G}}$ sürekli olsun.

$$\mathfrak{T}:(\mathfrak{F},\mathfrak{R})\to(\mathfrak{H},\mathfrak{Q})$$

aşağıdaki özellikleri sağlayan bir sürekli kendisinden yine kendisine olmayan esnek dönüşüm olsun ;

- $\mathfrak{T}$ proksimal $\mathfrak{L}-$kenarı korunması ve bir $\mathfrak{L}-$proksimal Kannan dönüşümüdür

öyleki

$$\mathfrak{T}((\mathfrak{F},\mathfrak{R}_0))\subseteq(\mathfrak{H},\mathfrak{Q}_0).$$

- $\tilde{\mathfrak{G}}((\mathfrak{ƀ}_1,\mathfrak{F}(\mathfrak{ƀ}_1)),\mathfrak{T}((\mathfrak{ƀ}_0,\mathfrak{F}(\mathfrak{ƀ}_0))))=\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$

ve

$$(\mathfrak{ƀ}_0,\mathfrak{F}(\mathfrak{ƀ}_0)),(\mathfrak{ƀ}_1,\mathfrak{F}(\mathfrak{ƀ}_1))\in\mathfrak{P}(\mathfrak{L}). \tag{4.2}$$

olacak şekilde $(\mathfrak{ƀ}_0,\mathfrak{F}(\mathfrak{ƀ}_0)),(\mathfrak{ƀ}_1,\mathfrak{F}(\mathfrak{ƀ}_1))\in(\mathfrak{F},\mathfrak{R}_0)$ vardır.

Bu takdirde $\mathfrak{T}$'nin $(\mathfrak{F},\mathfrak{R})$'da bir $(\mathfrak{ƀ}^*,\mathfrak{F}(\mathfrak{ƀ}^*))$ en yakın noktası vardır. Ayrıca, $\forall n\in\mathbb{N}$ için

$$\tilde{\mathfrak{G}}((\mathfrak{ƀ}_n,\mathfrak{T}(\mathfrak{ƀ}_{n-1}))=\tilde{\mathfrak{G}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

ile tanımlanan bir $\{(\mathfrak{k}_n, \mathfrak{I}(\mathfrak{k}_n)\}$ esnek dizisi $(\mathfrak{k}^*, \mathfrak{F}(\mathfrak{k}^*))$ esnek noktasına yakınsar. Üstelik, eğer $\mathfrak{L}$ 'de $(\mathfrak{F}, \mathfrak{R})$'nın herhangi iki nokta arasında bir yol varsa, o halde en yakın noktası tektir.

İspat: $(4,2)$'den, $\tilde{\mathfrak{O}}((\mathfrak{k}_1, \mathfrak{F}(\mathfrak{k}_1)), \mathfrak{T}((\mathfrak{k}_0, \mathfrak{F}(\mathfrak{k}_0)))) = \tilde{\mathfrak{O}}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q}))$ ve

$$(\mathfrak{k}_0, \mathfrak{F}(\mathfrak{k}_0)), (\mathfrak{k}_1, \mathfrak{F}(\mathfrak{k}_1)) \in \mathfrak{P}(\mathfrak{L}). \tag{4.3}$$

olacak şekilde $(\mathfrak{k}_0, \mathfrak{F}(\mathfrak{k}_0)), (\mathfrak{k}_1, \mathfrak{F}(\mathfrak{k}_1)) \in (\mathfrak{F}, \mathfrak{R}_0)$ vardır.

$\mathfrak{T}((\mathfrak{F}, \mathfrak{R}_0)) \subseteq (\mathfrak{H}, \mathfrak{Q}_0)$ olduğundan, $\mathfrak{T}((\mathfrak{k}_1, \mathfrak{F}(\mathfrak{k}_1))) \in (\mathfrak{H}, \mathfrak{Q}_0)$ ve

$$\tilde{\mathfrak{O}}((\mathfrak{k}_2, \mathfrak{F}(\mathfrak{k}_2)), \tilde{\mathfrak{O}}((\mathfrak{k}_1, \mathfrak{T}(\mathfrak{k}_1)))) = \tilde{\mathfrak{O}}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q})) \tag{4.4}$$

olacak şekilde $(\mathfrak{k}_2, \mathfrak{F}(\mathfrak{k}_2)) \in (\mathfrak{F}, \mathfrak{R}_0)$ vardır. $\mathfrak{T}$, (4.3) ve (4.4)'ün proksimal $\mathfrak{L}$ – Kenarı korunmasını kullanarak,

$(\mathfrak{k}_1, \mathfrak{F}(\mathfrak{k}_1)), (\mathfrak{k}_2, \mathfrak{F}(\mathfrak{k}_2)) \in \mathfrak{P}(\mathfrak{L})$ elde edilir. Bu işleme devam ederek, $(\mathfrak{F}, \mathfrak{R}_0)$'da her $n \in \mathbb{N}$ için

$$\tilde{\mathfrak{O}}((\mathfrak{k}_n, \mathfrak{T}(\mathfrak{k}_{n-1})) = \tilde{\mathfrak{O}}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q})) \tag{4.5}$$

ile

$$(\mathfrak{k}_n, \mathfrak{T}(\mathfrak{k}_{n-1})) \in \mathfrak{P}(\mathfrak{L}) \tag{4.6}$$

olacak şekilde bir $\{\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)\}$ esnek diziyi elde edilir.

Şimdi $\{\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)\}$, $(\mathfrak{F}, \mathfrak{R})$'da bir esnek Cauchy dizisi olduğunu gösterelim. $\{\mathfrak{k}_{n_0}, \mathfrak{F}(\mathfrak{k}_{n_0})\} = \{\mathfrak{k}_{n_0+1}, \mathfrak{F}(\mathfrak{k}_{n_0+1})\}$ öyleki $n_0 \in \mathbb{N}$ varsa, $(4,5)$'dan, $\{\mathfrak{k}_{n_0}, \mathfrak{F}(\mathfrak{k}_{n_0})\}$, $\mathfrak{T}$'nin bir en yakın noktasıdır. O halde her $n \in \mathbb{N}$ için $\{\mathfrak{k}_{n-1}, \mathfrak{F}(\mathfrak{k}_{n-1})\} \neq \{\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)\}$ olsun. Her $n \in \mathbb{N}$ için $\mathfrak{T}$ bir $\mathfrak{L}$ – proksimal Kannan esnek dönüşümü olduğundan

$$(\{\mathfrak{k}_{n-1}, \mathfrak{F}(\mathfrak{k}_{n-1})\}, \{\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)\}) \in \mathfrak{P}(\mathfrak{L}), \quad \tilde{\mathfrak{O}}((\mathfrak{k}_n, \mathfrak{T}(\mathfrak{k}_{n-1})) = \tilde{\mathfrak{O}}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q}))$$

ve

$$\tilde{\Phi}((\{\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})\}, \mathfrak{T}(\{\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)\})) = \tilde{\Phi}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q}))\ ,$$

yani

$$\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1}))) \preceq \zeta^n[\tilde{\Phi}((\mathfrak{k}_{n-1}, \mathfrak{F}(\mathfrak{k}_{n-1})), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1}))) \tilde{+} \tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)))]$$

$$\preceq \tilde{\Phi}((\mathfrak{k}_{n-1}, \mathfrak{F}(\mathfrak{k}_{n-1})), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})))$$

Tümevarım yoluyla, $\forall n \in \mathbb{N}$ için

$$\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1}))) \preceq \zeta^n \tilde{\Phi}((\mathfrak{k}_0, \mathfrak{F}(\mathfrak{k}_0)), (\mathfrak{k}_2, \mathfrak{F}(\mathfrak{k}_2))) = \zeta^n C \tag{4.7}$$

elde edilir.

$\zeta < (1/2)$ ise, $n \to \infty$ iken, $\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1}))) \to \pi((\mathfrak{a}, \mathfrak{F}(\mathfrak{a}), 0)$.

$p \geq 1$ olsun.

(i). $\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+(2_p+1)}, \mathfrak{F}(\mathfrak{k}_{n+(2_p+1)}))) = \tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})))$

$$\tilde{+}\tilde{\Phi}((\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})), (\mathfrak{k}_{n+2}, \mathfrak{F}(\mathfrak{k}_{n+2}))) \tilde{+} \tilde{\Phi}((\mathfrak{k}_{n+2_p}, \mathfrak{F}(\mathfrak{k}_{n+2_p})), (\mathfrak{k}_{n+2_p+1}, \mathfrak{F}(\mathfrak{k}_{n+2_p+1}))$$

$$\preceq \zeta^n C \dot{+} \zeta^{n+1} C \dot{+} \cdots \dot{+} \zeta^{n+2_p} C$$

$n, p \to \infty$ iken

$$= (\zeta^n \dot{+} \zeta^{n+1} \dot{+} \cdots \dot{+} \zeta^{n+2_p}) \to \pi((\mathfrak{a}, \mathfrak{F}(\mathfrak{a})), 0) \tag{4.8}$$

(ii). $\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+2_p}, \mathfrak{F}(\mathfrak{k}_{n+2_p}))) = \mathfrak{F}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})))$

$$\dot{+}\tilde{\Phi}((\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})), (\mathfrak{k}_{n+2}, \mathfrak{F}(\mathfrak{k}_{n+2}))) \dot{+} \tilde{\Phi}((\mathfrak{k}_{n+2_p-2}, \mathfrak{F}(\mathfrak{k}_{n+2_p-2})), (\mathfrak{k}_{n+2_p}, \mathfrak{F}(x_{n+2_p})))$$

$$\preceq \dot{+}\tilde{\Phi}((\mathfrak{k}_n, \mathfrak{F}(\mathfrak{k}_n)), (\mathfrak{k}_{n+1}, \mathfrak{F}(x_{n+1}))) \dot{+} \tilde{\Phi}((\mathfrak{k}_{n+1}, \mathfrak{F}(\mathfrak{k}_{n+1})), (\mathfrak{k}_{n+2}, \mathfrak{F}(\varkappa_{n+2})))$$

$$\dot{+}\tilde{\Phi}((\mathfrak{k}_{n+2_p-2}, \mathfrak{I}(\mathfrak{k}_{n+2_p-2})), (\mathfrak{k}_{n+2_p-1}, \mathfrak{F}(\mathfrak{k}_{n+2_p-1}))) \dot{+} \tilde{\Phi}((\mathfrak{k}_{n+2_p-1}, \mathfrak{F}(\mathfrak{k}_{n+2_p-1})), (\mathfrak{k}_{n+2_p}, \mathfrak{F}(\mathfrak{k}_{n+2_p})))$$

$$\dot{+}\tilde{\Phi}((\mathfrak{k}_{n+2_p-2}, \mathfrak{F}(\mathfrak{k}_{n+2_p-2})), (\mathfrak{k}_{n+2_p}, \mathfrak{k}(\varkappa_{n+2_p})))$$

$$\preceq \sum_{k=n}^{n+2_p-1} C\zeta^k \dot{+} \tilde{\Phi}((\mathfrak{k}_{n+2_p-2}, \mathfrak{F}(\mathfrak{k}_{n+2_p-2})), (\mathfrak{k}_{n+2_p}, \mathfrak{F}(\mathfrak{k}_{n+2_p}))) \tag{4.9}$$

$n, p \to \infty$ iken $\sum_{k=n}^{n+2_p-1} C\zeta^k \to \pi((\mathfrak{a}, \mathfrak{F}(\mathfrak{a})), 0)$ olduğunu biliyoruz. O halde

$$n, p \to \infty \text{ iken } \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-2})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p}))) \to \pi((\mathfrak{a}, \mathfrak{F}(\mathfrak{a})), 0).$$

gösterelim. (4.5)' dan,

$$\tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\varkappa_{n+2_p-2})), \mathfrak{T}((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})))) = \mathfrak{F}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q}))$$

(4.10)

$$\tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p})), \mathfrak{T}((\mathfrak{ß}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1})))) = \tilde{\mathfrak{G}}((\mathfrak{F}, \mathfrak{R}), (\mathfrak{H}, \mathfrak{Q})) \quad (4.11)$$

sonucuna varabiliriz.

Diğer yandan, (4.6) ten

$$((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})), (\mathfrak{ß}_{n+2_p}, \mathfrak{ß}(\mathfrak{ß}_{n+2_p}))) \in \mathfrak{P}(\mathfrak{L})$$

ve

$$((\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p})), (\mathfrak{ß}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1}))) \in \mathfrak{P}(\mathfrak{Q})$$

elde edilir. $\mathfrak{L}$ bir yöndirilmiş grafiği olduğundan, $\mathfrak{L}$ de $(\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1}))$

ve $(\mathfrak{ß}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1}))$ arasındaki bir yol vardır. Bu takdirde

$$((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})), (\mathfrak{F}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1}))) \in \mathfrak{P}(\mathfrak{L}) \quad (4.12)$$

$\mathfrak{T}$ 'nın bir $\mathfrak{L}-$proximal Kannan esnek dönüşüm olduğundan , (4.9) ve (4.10) dan,

$$\tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-2})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p}))) \preceq \zeta[\tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\varkappa_{n+2_p-1})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p})))$$

$$\dot{+}\ \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1})), (\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-2})))\ ]$$

$$\preceq \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\varkappa_{n+2_p}))) \dot{+} \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-2})), (\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})))$$

$$\dot{+} \mathfrak{M}((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p}))) \dot{+} \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p})), (\mathfrak{ß}_{n+2_p+1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p+1})))$$

$$= \zeta[2\tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-1})), (\mathfrak{ß}_{n+2_p}, \mathfrak{F}(\mathfrak{ß}_{n+2_p}))) \dot{+} \tilde{\mathfrak{G}}((\mathfrak{ß}_{n+2_p-2}, \mathfrak{F}(\mathfrak{ß}_{n+2_p-2})), (\mathfrak{ß}_{n+2_p-1}, \mathfrak{F}(\tilde{\Pi}_{n+2_p-1}))$$

$$\dot{+}\tilde{\mathfrak{D}}((\mathfrak{h}_{n+2_p},\mathfrak{F}(\mathfrak{h}_{n+2_p})),(\mathfrak{h}_{n+2_p+1},\mathfrak{F}(\mathfrak{h}_{n+2_p+1})))] \tag{4.13}$$

ve $n, p \to \infty$ iken

$$\tilde{\mathfrak{D}}((\mathfrak{h}_{n+2_p-2},\mathfrak{F}(\mathfrak{h}_{n+2_p-2})),(\mathfrak{h}_{n+2_p},\mathfrak{F}(\mathfrak{h}_{n+2_p}))) \preceq \zeta[C\zeta^{n+2_p-1} \dot{+} C\zeta^{n+2_p-2} \dot{+} C\zeta^{n+2_p} \to \pi((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),0)$$

elde edilir. (4.5)'den, $n, p \to \infty$ iken $\tilde{\mathfrak{D}}((\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n)),(\mathfrak{h}_{n+2_p},\mathfrak{F}(\mathfrak{h}_{n+2_p}))) \to \pi((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),0)$ olur. Bu takdirde $\{(\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n))\}$, $(\mathfrak{F},\mathfrak{R})$'da bir esnek Cauchy dizisidir. $(\mathfrak{F},\mathfrak{R})$ kapalı olduğundan, $n, p \to \infty$ iken $(\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n)) \to (\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))$ öyleki $(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)) \in (\mathfrak{F},\mathfrak{R})$ vardır. $\mathfrak{T}$'nin sürekliliğiyle, $n \to \infty$ iken $\mathfrak{T}((\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n))) \to \mathfrak{T}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)))$ elde edilir. $\tilde{\mathfrak{D}}$'nin sürekli olduğundan $n \to \infty$ iken

$$\tilde{\mathfrak{D}}((\mathfrak{h}_{n+2_p-2},\mathfrak{F}(\mathfrak{h}_{n+2_p-2})),(\mathfrak{h}_{n+2_p},\mathfrak{F}(\mathfrak{h}_{n+2_p}))) \to \tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),\mathfrak{T}(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))).$$

bulunur. (4.5) ile,

$$\tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),\mathfrak{T}(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))) = \tilde{\mathfrak{D}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

sonucuna varabiliriz. Böylece $(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))$, $\mathfrak{T}$'nin bir esnek yakın noktası ve her $n \in \mathbb{N}$ için

$$\tilde{\mathfrak{D}}((\mathfrak{h}_{n+1},\mathfrak{F}(\mathfrak{h}_{n+1})),\mathfrak{T}((\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n)))) = \tilde{\mathfrak{D}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

olarak tanımlanan bir $(\mathfrak{h}_n,\mathfrak{F}(\mathfrak{h}_n))$ esnek dizisi $(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))$ noktasına yakınsar.

Şimdi tekliği göstereceğiz. $(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)) \neq (\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*))$ iki farklı en yakın nokta olduğunu kabul edelim. O halde

$$\tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),\mathfrak{T}(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*))) = \mathfrak{F}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

$$\tilde{\mathfrak{D}}((\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)),\mathfrak{T}(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*))) = \tilde{\mathfrak{D}}((\mathfrak{F},\mathfrak{R}),(\mathfrak{H},\mathfrak{Q}))$$

Bundan dolayı $(\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)) \in \mathfrak{P}(\mathfrak{L})$ elde edilir, yani

$$\tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{h}^*))) \preceq \zeta[\tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{h}^*))) \dot{+} \tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)))]$$

$$= 2\zeta\tilde{\mathfrak{D}}((\mathfrak{h}^*,\mathfrak{F}(\mathfrak{h}^*)),(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)))$$

verilir. Böylece,

$$(1-2\zeta)\tilde{\mathfrak{D}}((\mathfrak{ß}^*,\mathfrak{F}(\mathfrak{ß}^*))\ ,\ (\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)))\preceq\pi((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),0)$$

olur.

Bu demek ki $1-2\zeta\le 0\Rightarrow\zeta\ge\frac{1}{2}$. Bu ise $\zeta<\frac{1}{2}$ olması ile çelişir. Böylece $\tilde{\mathfrak{D}}((\mathfrak{ß}^*,\mathfrak{F}(\mathfrak{ß}^*))\ ,\ (\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*)))=\pi((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),0)$, yani $(\mathfrak{ß}^*,\mathfrak{F}(\mathfrak{ß}^*))=(\mathfrak{g}^*,\mathfrak{F}(\mathfrak{g}^*))$ ve sonuç olarak en yakın noktası tektir.

Teorem 4.7 $\mathcal{K}$, $\left((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}}\right)$ esnek kompakt metrik uzayında bir öz esnek sürekli dönüşüm olsun ve $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))\neq(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))$ olmak üzere her $(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\in(\mathfrak{F},\mathfrak{R})$ için

$$\tilde{\mathfrak{D}}\left(\mathcal{K}^n(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathcal{K}^n(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\right)<\max\left\{\begin{array}{l}\tilde{\mathfrak{D}}((\mathfrak{ß},\mathfrak{I}(\mathfrak{ß})),(\mathfrak{g},\mathfrak{F}(\mathfrak{ß}))),\tilde{\mathfrak{D}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathcal{K}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß}))),\\ \tilde{\mathfrak{D}}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathcal{K}(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))),\\ \frac{1}{2}\left[\tilde{\mathfrak{D}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathcal{K}(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))+\tilde{\mathfrak{D}}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathcal{K}\tilde{\mathfrak{D}}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))\right]\end{array}\right\}\quad(4.14)$$

eşitsizliği sağlansın. O halde $\mathcal{K}$, $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$'da bir tek esnek sabit noktaya sahiptir.

İspat

$((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$ esnek kompakt metrik uzay ve

$$f((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathcal{K}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))=\tilde{\mathfrak{D}}((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})),\mathcal{K}(\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))$$

Olmak üzere f, $((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$'de bir parametrik skaler değerli fonksiyonu olsun. $\mathcal{K}$ esnek sürekli olduğundan, f de esnek süreklidir. Böylece $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})$ için f maksimum ve minimum sahiptir öyle ki

$$f((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))=\min\{f((\mathfrak{ß},\mathfrak{F}(\mathfrak{ß})))\mid\ (\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\mathfrak{D}})\}\quad(4.15)$$

Şimdi $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))$ $\mathcal{K}$'nin bir esnek sabit noktası olduğunu göstereceğiz.

$f(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))=\tilde{\mathfrak{D}}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\succ\left(\tilde{\pi}(\mathfrak{a},\mathfrak{a}),0\right)$ olsun. n pozitif tamsayı değer olduğunda ve $\mathfrak{M}=\mathfrak{M}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))$ için

$$f\left(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\right)=\tilde{\Phi}\left(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\right)=\tilde{\Phi}\left(\mathcal{K}^n(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}^n\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\right)$$

$$\prec \max\left\{\begin{array}{l}\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\ \ \tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\ \ \tilde{\Phi}(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathcal{U}))),\\ \frac{1}{2}\left[\tilde{\Phi}\left((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}^2(\mathfrak{a},\mathfrak{F}(\mathfrak{v}))\right)\dot{+}(\tilde{\pi}(\mathfrak{a},\mathfrak{a}),0)\right]\end{array}\right\}$$

$$\tilde{\Phi}(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathcal{U})))\prec \max\left\{f((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),f(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\frac{1}{2}\left[f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\dot{+}f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))\right]\right\}$$

ve

$$\frac{1}{2}\left[\tilde{\Phi}\left((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}^2(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\right)\dot{+}(\tilde{\pi}(\mathfrak{a},\mathfrak{a}),0)\right]$$

$$\prec \max\left\{f((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),f(\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\frac{1}{2}\left[f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\dot{+}f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))\right]\right\}.$$

elde edilir. (4.15) 'den dolayı

$$\max\left\{f((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))),\ \frac{1}{2}\left[f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\dot{+}f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))\right]\right\}=f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))$$

olup $f(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))))\prec f((\mathfrak{a},\mathfrak{F}(\mathfrak{a})))$ bulunur. Fakat bu (4.15) ile çelişir. Böylece, $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))$ $\mathcal{K}$ 'nin bir esnek sabit noktasıdır.

Şimdi $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))$ 'nun tek olduğunu gösterelim. Kabul edelim ki $(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))\neq(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))$ başka esnek nokta olsun öyle ki

$$\mathcal{K}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))=(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))\Rightarrow\mathcal{K}^2((\mathfrak{a}',\mathcal{K}(\mathfrak{a}')))=(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')),...,\mathcal{K}^n((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))=(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')).$$

$$\tilde{\Phi}\left((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))\right)=\tilde{\Phi}\left(\mathcal{K}((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\mathcal{K}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))\right)=\tilde{\Phi}\left(\mathcal{K}^n((\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\mathcal{K}^n((\mathfrak{a}',\mathcal{K}(\mathfrak{a}')))\right)$$

$$\prec \max\left\{\begin{array}{l}\tilde{\Phi}((\mathfrak{a},\mathfrak{I}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))),\ \tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\ \tilde{\Phi}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')),\mathcal{K}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))),\\ \frac{1}{2}\left[\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),\mathcal{K}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))))\dot{+}\tilde{\Phi}((\mathfrak{a}',\mathfrak{F}(\mathfrak{v}')),\mathcal{K}((\mathfrak{a},\mathfrak{I}(\mathfrak{a}))))\right]\end{array}\right\}$$

$$\prec \max\left\{\begin{array}{l}\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))),\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a},\mathfrak{F}(\mathfrak{a}))),\tilde{\Phi}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')),(\mathfrak{a}',\mathfrak{I}(\mathfrak{a}'))),\\ \frac{1}{2}\left[\tilde{\Phi}((\mathfrak{a},\mathfrak{I}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{I}(\mathfrak{a}')))\dot{+}\tilde{\Phi}((\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')),(\mathfrak{a},\mathfrak{F}(\mathfrak{a})))\right]\end{array}\right\}$$

$$\prec\tilde{\Phi}\left((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}'))\right)$$

Yukarıdaki eşitsizliği kullanarak,

$$\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))\prec\tilde{\Phi}((\mathfrak{a},\mathfrak{F}(\mathfrak{a})),(\mathfrak{a}',\mathfrak{F}(\mathfrak{a}')))$$

elde edilir. Bu bir çelişkidir, o halde $\mathcal{K}$ 'nin esnek sabit noktası tektir.

Örnek 4.8

$\mathfrak{h}$, $\mathfrak{K} = [0,\infty)$, $\mathfrak{P} = \mathbb{Q}^+$'de bir metrik ve $(\mathfrak{F},\mathfrak{P})$, $\mathfrak{K}$ üzerinde bir esnek küme olsun, öyle ki her $(\mathfrak{q},\mathfrak{F}(\mathfrak{q})) \in (\mathfrak{F},\mathfrak{P})$ için $(\mathfrak{q},\mathfrak{F}(\mathfrak{q})) = [0,\mathfrak{q}]$ olarak tanımlansın.

$$\tilde{\mathfrak{D}} : (\mathfrak{F},\mathfrak{P}) \times (\mathfrak{F},\mathfrak{P}) \to (\mathfrak{R}, \mathbb{R}^+ \cup \{0\})$$

$$\tilde{\mathfrak{D}}((\mathfrak{q},\mathfrak{I}(\mathfrak{q})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))) = (\max\{\mathfrak{q},\mathfrak{b}\}, \mathfrak{h}(\mathfrak{q},\mathfrak{b}))$$

esnek metrik olsun.

$\mathfrak{T}$ dönüşümü aşağıdaki gibi tanımlayalım:

$$\mathfrak{T} : ((\mathfrak{F},\mathfrak{P}),\tilde{\mathfrak{D}}) \to ((\mathfrak{F},\mathfrak{P}),\tilde{\mathfrak{D}})$$

$$\mathfrak{T}((\mathfrak{q},\mathfrak{I}(\mathfrak{q}))) = \left(\frac{1}{2}\mathfrak{q}, \mathfrak{F}\left(\frac{1}{2}\mathfrak{q}\right)\right) = \left[0,\frac{1}{2}\right]$$

Buradan

$$\tilde{\mathfrak{D}}(\mathfrak{T}(\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})))) = \tilde{\mathfrak{D}}\left(\left(\frac{1}{2}\mathfrak{q}, \mathfrak{F}\left(\frac{1}{2}\mathfrak{q}\right)\right),\left(\frac{1}{2}\mathfrak{b}, \mathfrak{F}\left(\frac{1}{2}\mathfrak{b}\right)\right)\right)$$

$$= \left(\max\left\{\frac{1}{2}\mathfrak{q},\frac{1}{2}\mathfrak{b}\right\}, \mathfrak{h}\left(\frac{1}{2}\mathfrak{q},\frac{1}{2}\mathfrak{b}\right)\right)$$

$$= \frac{1}{2}(\{\mathfrak{q},\mathfrak{b}\}, \mathfrak{h}(\mathfrak{q},\mathfrak{b}))$$

$$= \frac{1}{2}\tilde{\mathfrak{D}}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b})))$$

Şimdi n pozitif reel sayısı için

$$\tilde{\mathfrak{D}}(\mathfrak{T}^n(\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathcal{T}^n(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))) = \tilde{\mathfrak{D}}\left(\left(\frac{1}{2^n}\mathfrak{q}, \mathfrak{F}\left(\frac{1}{2^n}\mathfrak{q}\right)\right),\left(\frac{1}{2^n}\mathfrak{b}, \mathfrak{F}\left(\frac{1}{2^n}\mathfrak{b}\right)\right)\right)$$

$$= \left(\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\}, \frac{1}{2^n}|\mathfrak{q}-\mathfrak{b}|\right)$$

$$= \max\left\{\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\}, \left|\frac{1}{2^n}\mathfrak{q}-\frac{1}{2^n}\mathfrak{b}\right|\right\}$$

$$= \left\{\max\left\{\frac{1}{2^n}\mathfrak{q}\frac{1}{2^n}\mathfrak{b}\right\}, \ \left|\mathfrak{T}^n(\mathfrak{q})-\mathfrak{T}^n(\mathfrak{b})\right|\right\}$$

$$\prec \left(\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\}, \max\left\{|\mathfrak{q}-\mathfrak{b}|,|\mathfrak{q}-\mathfrak{T}(\mathfrak{q})|,|\mathfrak{b}-\mathfrak{T}(\mathfrak{b})|,\frac{1}{2}[|\mathfrak{q}-\mathfrak{T}(\mathfrak{b})| \dot{+} |\mathfrak{b}-\mathfrak{T}(\mathfrak{q})|]\right\}\right)$$

$$\prec \max\left\{\begin{array}{l}\left(\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\},\ |\mathfrak{q}-\mathfrak{b}|\right),\left(\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\},\ |\mathfrak{q}-\mathfrak{T}(\mathfrak{q})|\right),\\ \left(\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{b}-\mathfrak{T}(\mathfrak{b})|\right),\\ \frac{1}{2}\left[\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{q}-\mathfrak{T}(\mathfrak{b})|\dot{+}\max\left\{\frac{1}{2^n}\mathfrak{q},\frac{1}{2^n}\mathfrak{b}\right\},|\mathfrak{b}-\mathfrak{T}(\mathfrak{q})|\right]\end{array}\right\}$$

$$=\max\left\{\tilde{\Theta}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\tilde{\Theta}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\tilde{\Theta}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})),(\mathfrak{b},\mathfrak{F}(\mathfrak{b}))),\right.$$

$$\left.\frac{1}{2}[\tilde{\Theta}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}(\mathfrak{b},\mathfrak{F}(\mathfrak{b})))\dot{+}\tilde{\Theta}((\mathfrak{b},\mathfrak{F}(\mathfrak{b})),\mathfrak{T}((\mathfrak{b},\mathfrak{F}(\mathfrak{b}))))]\right\} \qquad (4.16)$$

$\mathfrak{T}$'nin (4.16) eşitsizliği sağladığı görülür. Böylece $\mathfrak{T}$ tek bir esnek sabit noktaya sahiptir.

Teorem 4.8

$\mathfrak{T}$, $((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})$ esnek kompakt metrik uzayının öz bir esnek sürekli dönüşüm olsun ve $(\mathfrak{f},\mathfrak{F}(\mathfrak{f}))\neq(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))$ olmak üzere her $(\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\in(\mathfrak{F},\mathfrak{R})$ için ve λ,α,δ negatif olmayan parametrikler öyle ki $2\lambda+\alpha+\delta<1$ olacak şekilde

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))$$

$$\prec\lambda\left[\frac{\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))\dot{+}\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))}{1\dot{+}\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},F(\mathfrak{g})),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{f},\mathfrak{I}(\mathfrak{f}))))\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{I}(\mathfrak{g})))}\right]$$

$$\dot{+}\alpha\left[\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\right]\dot{+}\delta\tilde{\Theta}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))$$

eşitsizliği sağlansın. O halde $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

İspat: Öncelikle $\mathcal{K}$ fonksiyonu, $((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})$ esnek kompakt metrik uzayında tanımlayalım, Her $(\mathfrak{f},\mathfrak{F}(\mathfrak{f}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})$ için

$$\mathcal{K}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})))=\mathfrak{h}((\mathfrak{f},\mathfrak{F}(\mathfrak{f})),\mathfrak{T}((\mathfrak{f},\mathfrak{F}(\mathfrak{f}))))$$

şeklinde tanımlansın. $\tilde{\Theta}$ ve $\mathfrak{T}$'nin esnek sürekli dolayı, $\mathcal{K}$ de esnek süreklidir. $((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})$ esnek kompakt olduğundan, $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))\in((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})$ bir esnek nokta vardır öyle ki

$$\mathfrak{F}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))) = \inf\{\mathcal{K}((\mathfrak{z},\mathfrak{F}(\mathfrak{z}))) \mid \quad (\mathfrak{z},\mathfrak{F}(\mathfrak{z})) \in ((\mathfrak{F},\mathfrak{R}),\tilde{\Theta})\}$$

(4.17)

$\mathcal{K}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))) \neq (\tilde{\pi}(\mathfrak{y},\mathfrak{y}),0)$ ise $\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))) \neq (\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$ olur. O halde

$$\mathcal{K}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))) = \mathfrak{h}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))$$

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))$$

$$\prec\lambda\left[\frac{\tilde{\Theta}((\mathfrak{y},\mathfrak{I}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),(\mathfrak{y},\mathfrak{F}(\mathfrak{y})))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}{1\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{p},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}\right]$$

$$\dot{+}\alpha\left[\tilde{\Theta}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{z},\mathfrak{F}(\mathfrak{z}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))+\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))\right]\dot{+}\delta\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{I}(\mathfrak{y}))))$$

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))\prec[\lambda+\alpha]\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))+[\lambda+\delta]\mathfrak{F}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))).$$

$$[1-(\lambda+\alpha)]\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))\prec[\lambda+\delta]\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))),$$

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{y}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))<\frac{[\lambda+\delta]\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}{1-(\lambda+\alpha)}.$$

$2\lambda+\alpha+\delta<1$ olduğundan,

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))\prec\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))).$$

elde edilir.

O halde $\mathcal{K}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\prec\mathcal{K}((\mathfrak{y},\mathfrak{T}(\mathfrak{y})))$ elde edilir. Bu (4.17)'nin özelliği çelişir. Bu taktirde $\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))=(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$ ve $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$, $\mathfrak{T}$ 'nin bir esnek sabit noktasıdır.

Şimdi $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$'nin tek olduğunu gösterelim. Kabul edelim ki $(\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))\neq(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$, $\mathfrak{T}$ 'nin başka esnek sabit nokta olsun. Bu taktirde

$$\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),(\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))=\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))).$$

$$\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),(\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))$$

$$\prec\lambda\left[\frac{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')))\tilde{\Theta}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))+\tilde{\Theta}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')),\mathfrak{T}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))))+\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),(\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')))}{1+\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))))\tilde{\Theta}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{R}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')),\mathfrak{T}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}'))))+\tilde{\Theta}((\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')),\mathfrak{T}((\mathfrak{y},\mathfrak{I}(\mathfrak{y}))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),(\mathfrak{y}',\mathfrak{F}(\mathfrak{y}')))}\right]$$

$$\dot{+}\alpha[\tilde{\Phi}((\wp',\mathfrak{F}(\wp')),\mathfrak{T}((\wp,\mathfrak{F}(\wp)))\dot{+}\tilde{\Phi}((\wp',\mathfrak{F}(\wp')),\mathfrak{T}((\wp',\mathfrak{F}(\wp'))))]\dot{+}\delta\tilde{\Phi}((\wp',\mathfrak{F}(\wp')),(\wp',\mathfrak{F}(\wp')).$$

$$\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))\prec\lambda\frac{\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))}{1\dot{+}\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))}\dot{+}\lambda\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp,\mathfrak{F}(\wp))\dot{+}\delta\tilde{\Phi}((\wp',\mathfrak{F}(\wp')),(\wp',\mathfrak{F}(\wp')).$$

Her a pozitif reel sayısı için, $a<\frac{a}{1+a\cdot a}$ olduğundan

$\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))<[\lambda+\alpha+\delta]\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),(\wp',\mathfrak{F}(\wp'))$ bulunur. $2\lambda+\alpha+\delta<1$ olduğundan, Bu bir çelişkidir. Böylece $(\dot{\varkappa},\mathfrak{I}(\dot{\varkappa}))$, $\mathcal{T}$ 'nin tek esnek sabit noktasıdır.

Teorem 4.10

$\mathfrak{T}$, $((\mathfrak{F},\Re),\tilde{\Phi})$ esnek kompakt metrik uzayının öz bir esnek sürekli dönüşüm olsun ve δ,β,λ negatif olmayan parametrikleri öyle ki $2\beta+\lambda+\delta\prec 1$olmak üzere her $(\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{q},\mathfrak{F}(\mathfrak{q}))\in(\mathfrak{F},\Re),(\mathfrak{k},\mathfrak{F}(\mathfrak{k}))\neq(\mathfrak{q},\mathfrak{F}(\mathfrak{q}))$ için

$\tilde{\Phi}(\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q}))))$

$$\prec\beta\left[\frac{\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})))\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q}))))\dot{+}\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{q},\mathfrak{F}(\mathfrak{q})))}{1\dot{+}\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{q},\mathfrak{I}(\mathfrak{q}))))\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q}))))\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Phi}((\mathfrak{k},\mathfrak{I}(\mathfrak{k})),(\mathfrak{q},\mathfrak{F}(\mathfrak{q})))}\right]$$

$$\dot{+}\lambda\left[\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{k},\mathfrak{I}(\mathfrak{k}))))\dot{+}\tilde{\Phi}((\mathfrak{q},\mathfrak{F}(\mathfrak{q})),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q}))))\right]\dot{+}\delta\tilde{\Phi}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{q},\mathfrak{F}(\mathfrak{q})))$$

eşitsizliği sağlansın. O halde $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

İspat: Bunun ispati yukarıdakinin aynısıdır. (4.17) özelliği kullanarak,

$$\tilde{\Phi}(\mathfrak{T}((\wp,\mathfrak{F}(\wp)))),\mathfrak{T}(\mathfrak{T}((\wp,\mathfrak{F}(\wp))))))\prec\frac{[2\beta+\delta]\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),\mathfrak{T}((\wp,\mathfrak{F}(\wp)))))}{1-\lambda}.$$

elde edilir. $2\beta+\lambda+\delta<1$ olduğundan,

$$\tilde{\Phi}(\mathfrak{T}((\wp,\mathfrak{F}(\wp)))),\mathfrak{T}(\mathfrak{T}((\wp,\mathfrak{F}(\wp))))))\prec\tilde{\Phi}((\wp,\mathfrak{F}(\wp)),\mathfrak{T}((\wp,\mathfrak{F}(\wp))))).$$

kolayca görünür. Böylece $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

Teorem 4.11

$\mathfrak{T}$, $((\mathfrak{F},\Re),\tilde{\Phi})$ esnek kompakt metrik uzayının öz bir esnek sürekli dönüşüm olsun ve $\delta,\beta,\lambda,\delta$ negatif olmayan parametrikler öyle ki $2\alpha+2\beta+\lambda+\delta<1$olmak üzere $(\mathfrak{k},\mathfrak{F}(\mathfrak{k}))\neq(\mathfrak{q},\mathfrak{F}(\mathfrak{q}))$ olacak şekilde her $(\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{q},\mathfrak{F}(\mathfrak{q}))\in(\mathfrak{F},\Re)$ için

$\tilde{\Phi}(\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))),\mathfrak{T}((\mathfrak{q},\mathfrak{F}(\mathfrak{q}))))$

$$\prec\alpha\left[\frac{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))}{1\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))}\right]$$

$$\dot{+}\beta\left[\frac{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))}{1\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))}\right]$$

$$\dot{+}\lambda\left[\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\right]\dot{+}\delta\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))$$

özellikleri sağlansın. Bu takdirde $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

İspat. Teorem 4.7 ve Teorem 4.8 gibi ispat yöntemi aynı,

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{T}(\mathfrak{y}))))))<\frac{[\alpha+2\beta+\delta]\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}{1-\alpha-\lambda}.$$

elde edilir. $2\alpha+2\beta+\lambda+\delta<1$ olduğundan,

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))))\prec\tilde{\Theta}((\mathfrak{y},\mathfrak{I}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{I}(\mathfrak{y}))))$$

olur. Böylece $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

Teorem 4.12

$\mathfrak{T}$, $((\mathfrak{F},\Re),\tilde{\Theta})$ esnek kompakt metrik uzayının bir öz esnek sürekli dönüşüm olsun ve her $(\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))\in(\mathfrak{F},\Re),\ (\mathfrak{k},\mathfrak{F}(\mathfrak{k}))\neq(\mathfrak{g},\mathfrak{F}(\mathfrak{g}))$ ve $\alpha+\beta+\xi+2[\eta+\delta]\leq 2$ olacak şekilde $\alpha,\beta,\xi,\eta,\delta$ negatif olmayan skaler için

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))$$

$$\prec\alpha\left[\frac{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))}{1\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{g}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{I}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{I}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))}\right]$$

$$\dot{+}\beta\left[\frac{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))}{\tilde{\Theta}((\mathfrak{k},\mathfrak{I}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\gamma,\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))}\right]$$

$$\dot{+}\xi\left[\frac{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))}{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\dot{+}\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{g},\mathfrak{F}(\mathfrak{g}))))}\right]$$

$$\dot{+}\eta\max\{\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})))),\tilde{\Theta}((\mathfrak{g},\mathfrak{F}(\mathfrak{g})),\mathfrak{T}((\mathfrak{k},\mathfrak{F}(\mathfrak{k}))))\}\dot{+}\delta\tilde{\Theta}((\mathfrak{k},\mathfrak{F}(\mathfrak{k})),(\mathfrak{g},\mathfrak{F}(\mathfrak{g})))$$

eşitsizliği sağlansın. O halde $\mathfrak{T}$ bir tek esnek sabit noktaya sahiptir.

İspat.

İlk olarak $\mathcal{K}$ fonksiyonu $(\mathfrak{F},\mathfrak{R})$'da aşağıdaki gibi tanımlayalım:

Her $(\mathfrak{z},\mathfrak{F}(\mathfrak{z}))\in(\mathfrak{F},\mathfrak{R})$ için

$$\mathcal{K}(\mathfrak{z},\mathfrak{F}(\mathfrak{z}))=\tilde{\Theta}((\mathfrak{z},\mathfrak{F}(\mathfrak{z})),\mathfrak{T}((\mathfrak{z},\mathfrak{F}(\mathfrak{z}))))$$

olsun. $\tilde{\Theta}$ ve $\mathfrak{T}$, $(\mathfrak{F},\mathfrak{R})$'da sürekli olduğundan $\mathcal{K}$ da $(\mathfrak{F},\mathfrak{R})$'da süreklidir.

$(\mathfrak{F},\mathfrak{R})$'nın esnek kompakt olmasından, $(\mathfrak{z},\mathfrak{F}(\mathfrak{z}))\in(\mathfrak{F},\mathfrak{R})$ vardır, öyle ki

$$\mathcal{K}(\mathfrak{v},\mathfrak{I}(\mathfrak{v}))=\inf\{\mathcal{K}((\mathfrak{z},\mathfrak{F}(\mathfrak{z}))) \mid \ (\mathfrak{z},\mathfrak{F}(\mathfrak{z}))\in(\mathfrak{F},\mathfrak{R})\} \tag{4.18}$$

olur.

Eğer $\mathcal{K}(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))\neq 0$ ise, $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))\neq\mathfrak{T}((\mathfrak{y},\mathfrak{T}(\mathfrak{y})))$ olur ve

$$\mathcal{K}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))=\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))$$

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{v}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))$$

$$\prec\alpha\left[\frac{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{v}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))}{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))}\right]$$

$$\dot{+}\beta\left[\frac{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}\right]$$

$$\dot{+}\xi\left[\frac{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{v}))))\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\tilde{\Theta}\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))}{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{v}))))\dot{+}\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))))\dot{+}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))}\right]$$

$$\dot{+}\max\{\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}(\mathfrak{h},\mathfrak{F}(\mathfrak{y}))),\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\}\dot{+}\delta\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{T}(\mathfrak{y}))))$$

elde edilir. Bu takdirde

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\prec\frac{\alpha}{2}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))$$

$$\dot{+}\frac{\beta}{2}\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{v}))))\dot{+}(\gamma/2+\delta+\eta)\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))$$

Yanı

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))[1-\alpha/2-\beta/2]\prec(\xi/2\dot{+}\delta\dot{+}\eta)\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))$$

$$\tilde{\Theta}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))),\mathfrak{T}^2((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))<(\xi/2\dot{+}\delta\dot{+}\eta)/[1-\alpha/2-\beta/2]\tilde{\Theta}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})),\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))$$

olur. Bu durumda $\mathcal{S}=(\xi/2\dot{+}\delta\dot{+}\eta)/[1-\alpha/2-\beta/2]\leq 1$ ve $\alpha+\beta+\xi+2[\eta+\delta]\leq 2$ ise

$$\mathcal{K}(\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y}))))\prec\mathcal{S}\mathcal{K}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))$$

elde edilir. Bu (4.18) ile çelişkir ve $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))=\mathfrak{T}((\mathfrak{y},\mathfrak{F}(\mathfrak{y})))$ olur. Böylece $(\mathfrak{y},\mathfrak{F}(\mathfrak{y}))$, $\mathfrak{T}$'nin bir esnek sabit noktasıdır.

Şimdi $(\wp, \mathfrak{F}(\wp))$'nun tek olduğunu gösterelim. $(\wp', \mathfrak{F}(\wp')) \neq (\wp, \mathfrak{F}(\wp))$, $\mathfrak{T}$'nin başka bir esnek sabit nokta olsun. Bu durumda

$$\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), (\wp', \mathfrak{F}(\wp')))$$

$$\prec \alpha \left[\frac{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))\tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{I}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))}{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{I}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))} \right]$$

$$\dot{+} \beta \left[\frac{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))\tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))\tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))}{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp, \mathfrak{F}(\wp))))} \right]$$

$$\dot{+} \xi \left[\frac{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp, \mathfrak{F}(\wp))))\tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))}{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp', \mathfrak{F}(\wp')))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))) \dot{+} \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))} \right]$$

$$\dot{+} \eta \max\{\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), \mathfrak{T}((\wp, \mathfrak{F}(\wp)))), \tilde{\Phi}((\wp', \mathfrak{F}(\wp')), \mathfrak{T}((\wp', \mathfrak{F}(\wp'))))\} \dot{+} \delta\tilde{\Phi}((\mathfrak{p}, \mathfrak{F}(\wp)), (\wp', \mathfrak{F}(\wp')))$$

olur. O halde

$$\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), (\wp', \mathfrak{F}(\wp'))) < (\beta/2 + \delta + \eta)\tilde{\Phi}((\wp, \mathfrak{F}(\wp)), (\wp', \mathfrak{F}(\wp')))$$

elde edilir. Fakat $\beta + 2[\delta + \eta] \leq 2$ olmasıyla çelişkidir. Böylece $(\wp, \mathfrak{F}(\wp))$, $\mathfrak{T}$'nin bir tek esnek sabit noktasıdır.

Sonuçlar.

1) Teorem 4.11'te $\alpha = \beta = \lambda = 0,\ \delta = 1$ koyarsak, esnek metrik uzaylar üzerinde esnek kompakt kümeler için Edelstein'in **[20]** sonucunu elde ederiz.
2) Teorem 4.8'de $\lambda = 0,\ \alpha = 1$ koyarsak, esnek metrik uzaylar üzerinde esnek kompakt kümeler için Edelstein'in **[20]** sonucunu elde ederiz.
3) Teorem 4.10'de $\beta = 0,\ \lambda = 1$ koyarsak, esnek metrik uzaylar üzerinde esnek kompakt kümeler için Edelstein'in **[20]** sonucunu elde ederiz.

5. SONUÇ VE ÖNERİLER

Bu tez çalışmasında, esnek kompakt metrik uzaylar göz önüne alınarak, esnek kompakt metrik uzaylar için Edelstein sabit nokta teoreminin bazı genellemeleri elde edilmiştir. Ayrca teorinin uygun bir örnek ile desteklendiği görülmektedir. Böylece bu esnek kompakt metrik uzayda bazı sabit nokta teoremlerini ispatlamaya yönelik altyapı oluşturması açısından esnek metriğin kullanılmasıyla esnek dizi, esnek

yakınsak dizi ve esnek Cauchy dizisi gibi tanımlar verilmiştir. Esnek küme teorisinin tanımlanmasından bugüne cebirsel, topoloji ve uygulama alanı başta olmak üzere farklı alanlarda gelişmesini devam etmektedir.

Bu çalışmasında bazı sabit nokta teoremleri esnek metrik uzaylarda ispat edilmiştir. Bu teoremleri ispatlamaya yönelik bazı yardımcı tanım, teorem ve lemmalar da verilmiştir. Aynı evren üzerinde tanımlı bir esnek sabit nokta ve esnek yakın nokta olan ilişkisi ortaya konulmuştur

Aynı zamanda bu tez çalışmasında elde edilen sonuçların dışında bazı sabit nokta teoremi ifade ve ispat edilebilir. Ayrıca esnek yakın nokta teoremleri için kullanılan fonksiyonlara esnek sürekli olmaları veya esnek sürekli olmamaları gibi özellikler sunacak çalışmalar için temel oluşturulmuştur.

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